Properties of Matter

Part – A

1. **What is elasticity?**
The property of the body to regain its original shape or size, after the removal of deforming force is called elasticity

2. **Define Stress and Strain with its unit? What are its types?**
The deforming force applied per unit area of the body is called stress. There are three types: Longitudinal stress, tangential stress and bulk stress
$Stress = \frac{Force}{Area} Nm^{-2}$
Strain is the ratio of change in dimension to its original dimension. There are three types: longitudinal strain, angle of shear and bulk strain
$Strain = \frac{Change\ in\ dimension}{Original\ dimension}\ No\ unit$

3. **State Hooke's law?**
Within elastic limit, the stress developed in the body is directly proportional to strain produced in it.

$$\frac{Stress}{Strain} = E\ (constant)$$

4. **Define Young's modulus of elasticity?**
Within elastic limit, $Young's Modulus\ (Y) = \frac{Longitudional\ Stress}{Longitudional\ Strain} = \frac{Fl}{AL} Nm^{-2}$

5. **Define Rigidity Modulus of elasticity?**
Within elastic limit, $Rigidity\ Modulus\ (N) = \frac{Tangential\ \ Stress}{Shearing\ Strain} = \frac{F}{A\theta} Nm^{-2}$

6. **Define Bulk Modulus of elasticity?**
Within elastic limit, $Bulk\ Modulus\ (K) = \frac{Volume\ \ Stress}{Volume\ \ Strain} = \frac{-PV}{dv} Nm^{-2}$

7. **What is beam?**
A beam is a rod or bar of uniform cross section whose length is very much greater than its other dimensions.

8. **Define Poisson's ratio?**
Within elastic limit, the ratio of lateral strain to the longitudinal strain is said to be Poisson's ratio. i.e.,$Poission's\ ratio\ (\sigma) = \frac{Lateral\ strain\ (\beta)}{Longitudional\ Strain(\alpha)} = \frac{-(D-d)L}{Dl}$
Practically, Poisson's ratio lies between 0 and ½

9. **Define neutral surface and neutral axis?**
In the middle of the beam, there is a layer which is not elongated the beam or compressed due to bending of the beam. The layer is called the neutral surface and the line at which the neutral layer intersects the plane of bending is called the 'neutral axis'

10. Explain bending moment of a beam?

The moment of the couple due to the restoring couple which balances the external couple due to the applied load is called bending moment

11. How does temperature and impurity in the material affect the elasticity?

(i) In general rise in temperature decreases elasticity. In certain rare cases like invar steel, elasticity is unaffected by any change in temperature

(ii) Addition of impurity atoms distorts the lattice structure of the base metal which decrease its elastic property. Instead if the impurity atoms have similar atomic radii and electronic structure, then the elastic property increases or decreases depending upon the impurity elements are more elastic or plastic.

12. What is I shape Girder? Explain its advantages?

A girder is a metallic beam supported at its two ends by pillars or on opposite walls. It should be so designed that it should not bend too much or break under its own weight. The cross section of beam is in the form of letter 'I'

Advantages:

(i) As the upper & bottom layer are subjected to maximum stress more material must be needed to withstand strain. Hence material is removed around the stress region of the neutral axis.

(ii) Iron girders used in buildings are made of I section

(iii) I type of cross section provides a high bending moment and a lot of material is saved

(iv) I form of girders are made of steel as it has high young's modulus

13. What is cantilever?

It is a beam fixed horizontally at one end and loaded at the other end.

14. Define tensile strength?

It is the maximum value of tensile stress withstand by the material before fracture under a steady load

15. Define safety factor?

The ratio between ultimate tensile stress to the working stress is called the safety factor.

16. When a wire is bent back and forth, it becomes hot, why?

When a wire is bent back and forth, heat is generated due to the area of hysteresis and frictional force. Hence it becomes hot.

17. What do you infer from stress - strain diagram?

- It is used to determine the elastic strength, yield strength and tensile strength of metals
- It is used to estimate the working stress and safety factor of an engineering material. The lower value of the safety factor are adopted to keep the structure for long life.
- The area under the curve in elastic region gives the energy required to deform elastically. Whereas the area under the curve in ultimate tensile strength gives the energy required to deform plastically.
- It is also used to identify the ductile and brittle materials

18. **Define moment of a force, couple and torque?**

    The moment of force about a point is defined as the product of magnitude of the force and the perpendicular distance from the point to the line of action of force..

    A **couple** constitutes a pair of two equal and opposite forces acting on the body, in such a way the lines of action of the to forces are not in the same straight line

    Torque is a rotating force and is equal to the moment of the couple. Torque is the produce of the forces forming couple and the perpendicular distance between two opposite forces

19. **Define Torsional stress?**

    The shear stress setup in the shaft when equal and opposite torques are applied to the ends of a shaft about its axis is called torsional stress.

20. **Define yield point?**

    If the external stress applied is very large, then the body losses its elastic behaviour, even after the removal of stress. The point at which the body losses its elasticity is called yield point.

21. **Define elastic fatigue?**

    If a body is continuously subjected to stress (or) strain, it gets fatigued called as elastic fatigue.

22. **What is a torsion pendulum? What are its uses?**

    A circular metallic disc suspended using a thin wire that executes torsional oscillation is called torsional pendulum. It is used to determine rigidity modulus of wire, moment of inertia of disc and moment of inertia of an irregular body.

PART – B

1. **Deduce an expression for the couple to produce a unit twist in along cylindrical wire fixed at one end. How is it used to determine the rigidity modulus of the wire?**

   (or)

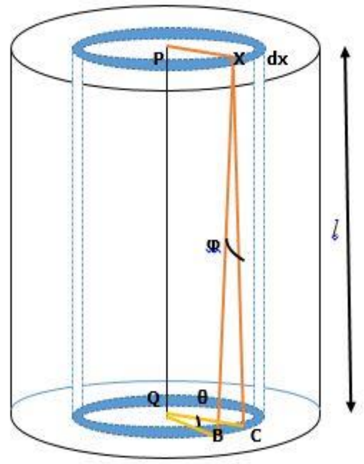
   Derive an expression for the period of oscillation of torsion pendulum. How is used to determine the torsional rigidity of the wire?

Twisting couple on a wire

Consider a cylindrical wire of length ***l*** and radius **r** fixed at one end. It is twisted through an angle θ by applying couple to its lower end. Now, the wire is said to be under torsion. Due to elastic property of the wire, an internal restoring couple is setup inside the wire. It is equal and opposite to the external twisting couple. The cylinder is imagined to consist of a large number of thin hollow cylinders. Consider one such cylinder of radius **x** and thickness **dx. AB** is a line parallel to **PQ** on the surface of this cylinder. As the cylinder is twisted, the line **AB** is shifted to **AC** through an angle **BAC = ϕ**

Shearing Strain = ϕ

Angle of twist at the free end = θ



From the figure, **BC** = x θ = l θ (or) $\phi = \frac{x\theta}{l}$

Rigidity modulus (n) = $\frac{\text{Shearing Stress}}{\text{Shearing Strain}}$

∴Shearing stress = n x Shearing strain = $n\phi = \frac{nx\theta}{l}$

But, Shearing stress = $\frac{\text{Shearing Force}}{\text{Area over which the force acts}}$

Shearing Force = Shearing stress x area over which the force acts

Area over which the force acts is $\pi(x+dx)^2 - \pi x^2 = 2\pi x\, dx$ (neglecting dx^2)

Hence, shearing force $F = \frac{nx\theta}{l} 2\pi x dx$

Twisting couple on a wire

shearing force $F = \frac{2\pi n\theta}{l} x^2 dx$

∴Moment of this force about the axis **PQ** of the cylinder = Force x perpendicular distance

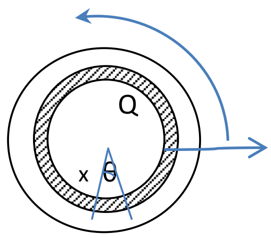
$$= \frac{2\pi n\theta}{l} x^2 dx \times x$$

$$= \frac{2\pi n\theta}{l} x^3 dx$$

The moment of the force acting on the entire cylinder of radius $\boldsymbol{r}$ is obtained by integrating the above expression between the limits $x = r$ and $x = 0$

Hence, twisting couple $C = \int_0^r \frac{2\pi n\theta}{l} x^3 dx$

$$\frac{2\pi n\theta}{l} \int_0^r x^3 dx = \frac{2\pi n\theta}{l} \left[\frac{x^4}{4}\right]_0^r$$



$$\therefore C = \frac{\pi n r^4 \theta}{2l}$$

In the above equation, if θ = 1 radian, then, we get

Twisting couple per unit twist $\boxed{C = \frac{\pi n r^4}{2l}}$

This twisting couple required to produce a twist of unit radian in the cylinder is called the torsional rigidity for material of the cylinder

Torsional Pendulum

A torsional pendulum is a pendulum performing torsional oscillations. It is used to find the rigidity modulus of the material of the wire and moment of inertia of a given disc

Description

A torsional pendulum consists of a metal wire suspended vertically with the upper end fixed. The lower end of the wire is connected to the center of a heavy circular disc as shown in figure. When the disc is rotated by applying a twist, the wire is twisted is twisted through an angle θ.

Then, the restoring couple setup in the wire = **Cθ** where **C** is the couple per unit twist.

If the disc is released, it oscillates with angular velocity $\frac{d\theta}{dt}$ in the horizontal plane about the axis of the wire. These oscillations are known as **torsional oscillations.** If $\frac{d^2\theta}{dt^2}$ is the angular acceleration produced in the disc and **I** its moment of inertia about the axis of the wire then,

Applied couple $= I\frac{d^2\theta}{dt^2}$

At equilibrium position, Applied couple = Restoring couple

i.e., $I\frac{d^2\theta}{dt^2} = C\theta$

(or) $\frac{d^2\theta}{dt^2} = -\frac{C}{I}\theta$

This equation represents simple harmonic motion which shows that angular acceleration is proportional to angular displacement θ and is always directed towards the mean position. Hence, the motion of the disc being *simple harmonic motion,*

The time period of the oscillation is given by $T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$

$$= 2\pi\sqrt{\frac{\theta}{\frac{C}{I}\theta}}$$

(or) $$\boxed{T = 2\pi\sqrt{\frac{I}{C}}}$$

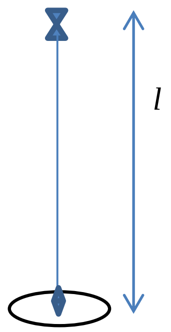
Determination of Rigidity modulus of the wire

A circular disc is suspended by a thin wire, whose rigidity modulus is to be determined. The top end of the wire is fixed tightly in a vertical support. The disc is then rotated about its center through a small angle and set it free. It executes torsional oscillations. The time taken for 20 complete oscillations is noted. The experiment is repeated and the mean time period (T) of oscillation is found out.

The length *l* of the wire is measured. This length is then changed and the experiment is repeated for five or six different lengths of wire are measured and tabulated. The disc is removed and its mass and diameter are measured

The time period of oscillation is $\mathbf{T = 2\pi \sqrt{\frac{I}{C}}}$

(or) $T^2 = 4\pi^2 \frac{I}{C}$



Substituting couple per twist $\mathbf{C = \frac{\pi n r^4}{2l}}$

$$T^2 = 4\pi^2 \frac{I}{\frac{\pi n r^4}{2l}}$$

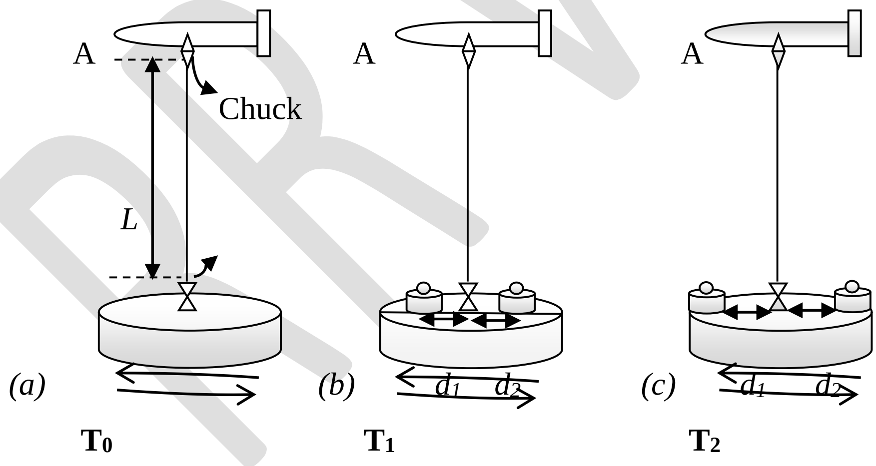
(or) $$\boxed{n = \frac{8\pi I}{r^4}\left[\frac{l}{T^2}\right]}$$

Where **I** is moment of inertia of circular disc which is equal to $\frac{MR^2}{2}$

M- Mass of the circular disc; R – Radius of the disc

2. Discuss the relevant theory of rigidity modulus of torsion pendulum using two symmetrical masses and determinate its moment of inertia of the circular disc and rigidity modulus of the suspended wire experimentally?

The torsion pendulum consists of a steel or brass wire with one end fixed in an adjustable chuck and the other end to the center of a circular disc as shown in figure (a)



The experiment consists of three parts: First the disc is set into torsional oscillations without any cylindrical masses on the disc. The mean period of oscillation '$\mathbf{T_0}$' is found out. Now, $\mathbf{T_0 = 2\pi \sqrt{\frac{I_0}{C}}}$

Where $\mathbf{I_0}$ is moment of the inertia of the disc about the axis of the wire

$T_0^2 = 4\pi^2 \frac{I_0}{C}$ (1)

Two equal cylindrical masses (each mass *m* equal to 200 gms) are placed symmetrically along a diameter of the disc at equal distance d_1 on the two sides of the center of the disc as shown in *Figure (b)*

Mean time period of oscillation T_1 is found (Fig(b)). Then, $T_1 = 2\pi\sqrt{\frac{I_1}{C}}$ (or) $T_1^2 = 4\pi^2 \frac{\mathbf{I_1}}{\mathbf{C}}$ (2)

Here, **I_1 -** Moment of inertia of the whole system about the axis of the wire

C - Couple per unit twist

i – Moment of inertia of each mass about the axis passing through its centre

Then, by the parallel axis theorem, the moment of inertia of the whole system is

$$\mathbf{I_1 = I_0 + 2\textit{i} + 2\textit{m}\textit{d}_1^2} \quad (3)$$

Substituting the value of I_1 in equation (2),

$$T_1^2 = \frac{4\pi^2}{C}(I_0 + 2i + 2md_1^2) \quad (4)$$

Now, two masses are placed symmetrically at equal distances d_2 *from the axis of the wire as shown in figure(c)*

Mean time period of oscillation T_2 is found. In this case, $T_2 = 2\pi\sqrt{\frac{I_2}{C}}$ (or) $T_2^2 = 4\pi^2 \frac{\mathbf{I_2}}{\mathbf{C}}$

$$T_2^2 = \frac{4\pi^2}{C}(I_0 + 2i + 2md_2^2) \quad \text{[By parallel axis theorem]} \quad (5)$$

Now, $\mathbf{I_2 - I_1 = 2m\,(d_2^2 - d_1^2)}$

Eqn. (5) – Eqn. (4) » $(T_2^2 - T_1^2) = \frac{4\pi^2}{C} 2m\,(d_2^2 - d_1^2)$

(or) $(T_2^2 - T_1^2) = \frac{4\pi^2}{C}(I_2 - I_1)$ (6)

Eqn. (1) ÷ Eqn. (6) -» $\frac{T_0^2}{T_2^2 - T_1^2} = \frac{I_0}{I_2 - I_1}$ (7)

Substituting the values of $(\mathbf{I_2 - I_1})$ in eqn. (7), we get, $\frac{T_0^2}{T_2^2 - T_1^2} = \frac{I_0}{2m(d_2^2 - d_1^2)}$ (or)

$$\mathbf{I_0} = \frac{2m(d_2^2 - d_1^2)T_0^2}{T_2^2 - T_1^2}$$

Thus, the moment of inertia of the disc about the axis of rotation is calculated by substituting the value of

T_0, T_1, T_2, d_2 *and* d_1 in the above formula.

Calculation of rigidity modulus of the material of the wire

We know that restoring couple per unit twist **C** = $\frac{\pi n r^4}{2l}$ (8)

Substituting the value of **C** in expression (6) we have, $(T_2^2 - T_1^2) = \frac{4\pi^2}{\frac{\pi n r^4}{2l}} (d_2^2 - d_1^2)$

(or) $(T_2^2 - T_1^2) = \frac{4\pi^2 \times 2l}{\pi n r^4} 2m (d_2^2 - d_1^2)$

$$\boxed{\mathbf{n = \frac{16\,\pi\, l\, m(d_2^2 - d_1^2)}{(T_2^2 - T_1^2) r^4}}} \quad N/m^2$$

Using the above relation, the rigidity modulus of wire is determined.

3. Give the theory of torsion pendulum and describe a method of find the moment of inertia and rigidity modulus of an irregular body?

Here the torsion pendulum consists of a cradle (c) which is in form of a horizontal circular disc fixed to a rectangular metallic frame. The cradle is suspended from a fixed end with the help of wire as shown in figure.

There is a concentric circular groove at the centre of the disc. So that any object for which the moment of inertia to be found can be placed over it.

Initially, the cradle alone is rotated and set into torsional oscillation and the time period of oscillation (T) is found

i.e., $T = 2\pi\sqrt{\frac{I}{C}}$

(or) $T^2 = 4\pi^2 \frac{I}{C}$ (1)

Where I is the moment of inertia of the cradle.

Now, the regular body is placed over the cradle and is allowed to produce torsional oscillations and time period of oscillation (T_1) is found.

i.e., $T_1 = 2\pi\sqrt{\frac{I + I_1}{C}}$

(or) $T_1^2 = 4\pi^2 \frac{I + I_1}{C}$ (2)

Where I_1 is the moment of inertia of the regular body (Known value)

Now, the regular body is removed from the cradle and an irregular body is placed over the cradle and is allowed to produce torsion oscillations. The time period of oscillation (T_2) is found.

i.e., $T_2 = 2\pi\sqrt{\frac{I + I_2}{C}}$

(or) $T_2^{\ 2} = 4\pi^2 \frac{I + I_2}{C}$ (3)

Where I_2 is the moment of inertia of irregular body (unknown)

From Eqns. (1), (2) and (3)

$$\frac{T_1^2 - T^2}{T_2^2 - T^2} = \frac{\frac{4\pi^2}{c}\left[I + I_1 - I\right]}{\frac{4\pi^2}{c}\left[I + I_2 - I\right]}$$

(or) $$I_2 = I_1\left(\frac{T_2^2 - T^2}{T_1^2 - T^2}\right)$$

Thus by knowing the moment of inertia of regular body, time periods T, T_1 and T_2, the moment of inertia is determined.

Rigidity Modulus

Subtract Eqn. (2) from (1)

$$T_1^2 - T^2 = 4\pi^2 \frac{I_1}{C}$$

We know that $C = \frac{\pi n r^4}{2l}$

Hence $T_1^2 - T^2 = 4\pi^2 \frac{I_1 \times 2l}{\pi n r^4}$

(or) $$n = \frac{8\pi I_1 l}{\left(T_1^2 - T^2\right) r^4}$$

Using this the rigidity modulus of the irregular body is determined.

4. Derive an expression for the internal bending moment of a beam in terms of radius of curvature?

Bending moment of the beam

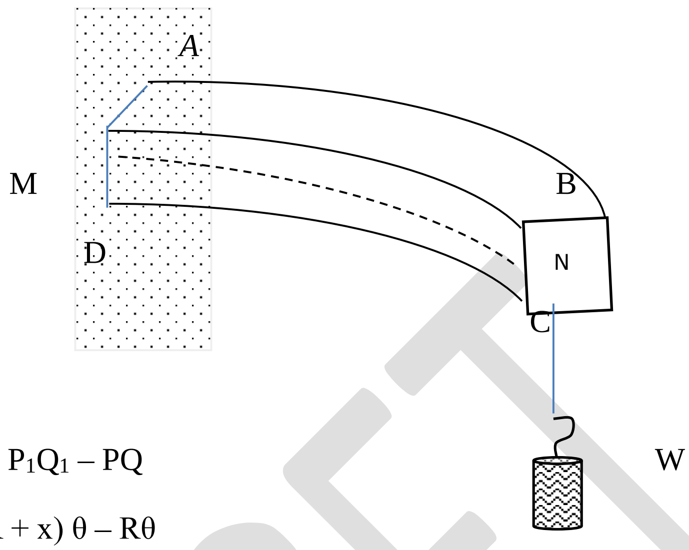
Consider a portion ABCD of the bent beam as shown in fig. P and Q are two points on the neutral axis MN. R is the radius of curvature of the neutral axis and θ is the angle subtended by bent beam at its centre of curvature O. i.e., ∠ POQ = θ

Consider two corresponding points P_1 and Q_1 on a parallel layer at a distance '*x*' from the neutral axis.

From the fig. PQ = R x θ …………….. (1)

Corresponding length on the parallel layer

$P_1Q_1 = (R + x)\,\theta$ ……………. (2)



Increase in length of $P_1Q_1 = P_1Q_1 - PQ$

$= (R + x)\,\theta - R\theta$

$= R\theta + x\,\theta - R\theta = x\,\theta$

Before bending $P_1Q_1 = PQ$

Longitudinal strain produced = $\frac{Increase\ in\ length}{original\ length}$

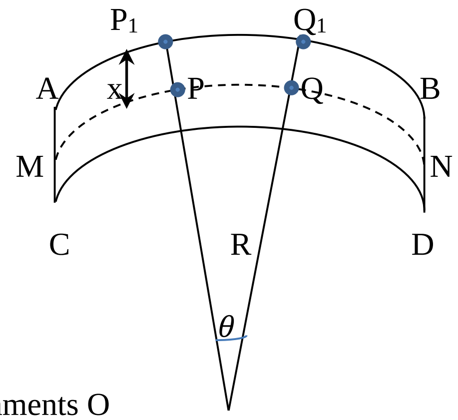
$= \frac{x\theta}{R\theta} = \frac{x}{R}$ …………… (3)

If Y is the young's modulus of the material

$$Y = \frac{Longitudinal\ stress}{Longitudinal\ strain}$$

i.e., Longitudinal stress = Y x Longitudinal strain

$= \frac{x}{R}\,Y$ …………….(4)



If δA is area of cross- section of the filament then,

Force acting on the area δA = stress x area

$= \frac{Y.x}{R}\,\delta A$

Moment of this force about the neutral axis MN

$= \frac{Y.x^2}{R}\,\delta A$

The sum of the moments of forces acting on all the filaments O

$$= \sum \frac{Y.x^2}{R}\,\delta A$$

$$= \frac{Y}{R} \sum \delta A. x^2$$

$\sum \delta A. x^2$ = I is called geometrical moment of inertia of the cross section of the beam

$$= \frac{YI}{R}$$

The sum of the moments of forces acting on all the filaments is the internal bending moment which comes in to play due to elasticity

Thus, internal bending moment of a beam = $\frac{YI}{R}$

For a rectangular beam of breadth (b) and thickness (d), I = $\frac{bd^3}{12}$

For a beam of circular cross section I = $\frac{\pi r^4}{4}$ where r is the radius of the rod

5. What is cantilever? Obtain an expression for depression at the loaded end of a cantilever whose other end is fixed assuming that its own weight is not effective in bending? Also explain experimental verification of cantilever?

It is a beam fixed horizontally at one end and loaded at the other end.

AB is the neutral axis of the cantilever of length '***l***' fixed at the end '**A**' and loaded at the free end '**B**' horzontally by a weight ' **W**'

The end B is depressed to **B′** (as shown in fig.)

BB' represents the vertical depression of the free end.

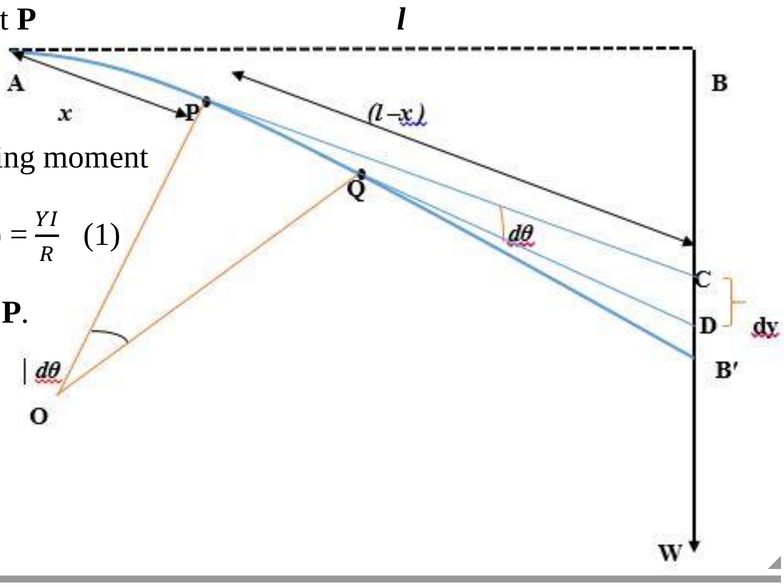
Consider the section of the cantilever **P** at distance '**x**' from the fixed end **A**. It is at a distance (l – x) from the loaded end **B′**

Considering the equilibrium of the portion **PB′**, there is a force of reaction **W** at **P**.

∴ External bending moment = **W** x **PB′** = **W (*l* – x)**

Internal bending moment of the cantilever = $\frac{YI}{R}$

Where Y – Young's modulus of cantilever, I - geometrical moment of inertia of the cross section & R – radius of curvature of neutral axis at **P**



In the equilibrium position,

External bending moment = Internal bending moment

$$W (l – x) = \frac{YI}{R} \quad (1)$$

Q is another point at the distance **dx** from **P**.

i.e., PQ = dx

'O' is the centre of curvature of the arc

PO = R ;∠ POQ = dθ

Then dx = R dθ (2)

The tangents are drawn at P & Q meeting the vertical line BB' at C and D

Vertical depression CD = dy = (l – x) dθ (3)

From equations (2) & (3)

$\frac{dx}{dy} = \frac{Rd\theta}{(l-x)d\theta} \quad = \frac{R}{(l-x)}$

(or) R = $\frac{(l-x)dx}{dy}$ (4)

Substituting the value of R in equation (1), we have

W(l – x) = $\frac{Y\ I}{\frac{(l-x)dx}{dy}}$

W(l – x) = $\frac{Y\ I\ dy}{(l-x)dx}$ (5)

(or) dy = $\frac{W}{YI}(l-x)(l-x)dx$

dy = $\frac{W}{YI}(l-x)^2dx$ (6)

Total depression Y(=BB') at the free end is y = $\int_0^l \frac{W}{YI}(l-x)^2dx$ (7)

$$y = \frac{W}{YI}\int_0^l (l-x)^2dx$$

$$y = \frac{W}{YI}\int_0^l (l^2 + x^2 - 2lx)dx$$

$$y = \frac{W}{YI}\left[l^2x + \frac{x^3}{3} - \frac{2lx^2}{2}\right]_0^l$$

$$y = \frac{W}{YI}\left[l^3 + \frac{l^3}{3} - l^3\right] \quad = \frac{W}{YI} \times \frac{l^3}{3}$$

(or) $$y = \frac{Wl^3}{3YI} \quad (8)$$

The young's modulus of the cantilever is determined using the value of the depression produced in the cantilever.

The depression at the free end of the single cantilever is y = $\frac{Wl^3}{3YI}$ (or) Y = $\frac{Wl^3}{3Iy}$ (9)

For a beam of a rectangular cross section (I) = $\frac{bd^3}{12}$

Where *b* is breadth and *d* the thickness of the beam.

The weight (W) = Mg

Where M is the mass suspended at the free end and g is the acceleration due to gravity

Hence Y = $\frac{Mgl^3}{3\frac{bd^3}{12}y}$

(or) Y = $\frac{4Mgl^3}{bd^3y}$

From which Y is determined experimentally

Experimental Determination of young's modulus by cantilever depression

Description

It consists of beam clamped rigidly at one end on the table. The weight Hanger is suspended at the other end of the beam through a small groove on the beam as shown in figure. A pin is fixed at the free end of the beam by means of wax. A microscope is placed in front of this arrangement for measuring the variation of height of the pin

Procedure

The weight hanger is kept hanged in a dead load position without slotted weights. The microscope is adjusted and the tip of the pin is made to coincide with the horizontal cross wire. The reading in the vertical scale of the microscope is noted. This reading is repeated by increasing the values of W in steps of 50gms and noted in increasing load. Then, the experiment is repeated by removing the weights step by step and noted in decreasing load

From these observation, the mean depression y corresponding to each value of M is obtained. The length of the beam (*l)* breadth of the beam (b) using vernier caliper and thickness of the beam (t) using screw gauge are noted

The young's modulus of the cantilever is determined by the relation $Y = \frac{4Mgl^3}{bd^3y}$

Sl.No	**Load (M)**	**Microscopic readings**			**Depression (Y)**
		Increasing Load	**Decreasing Load**	**Mean**	
1	W				
2	W+50				
3	W+100				
4	W+150				
5	W+200				

6. **(i) derive an expression for the elevation at the centre of cantilever which is loaded at both ends**
   (iii) Describe an experiment to determine Young's modulus of a beam by uniform bending

Definition

The beam is loaded uniformly on its both ends, the bend beam forms an arc of an circle. The elevation is produced in the beam. This type bending is known as uniform bending

Consider a beam (or bar) AB arranged horizontally on two knife edges C & D symmetrically so that AC = BD = a

As shown in figure.

The beam is loaded with equal weights 'W' at each ends A and B.

The reactions on the knife edges at C and D are equal to W and they are acting vertically upwards.

The external bending moment on the part AF of the beam is

W x AF - W x CF = W (AF – CF)

W x AC = W x a = W a (1)

Internal bending moment = $\frac{YI}{R}$ (2)

Where Y – Young's modulus of the material of the bar, I - geometrical moment of inertia of the cross section of a beam & R – radius of curvature of bar at F

In the equilibrium position, External bending moment = Internal bending moment

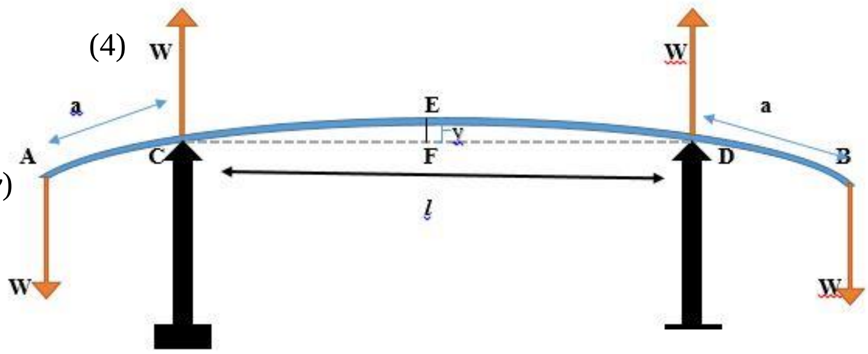
Hence, W a = $\frac{YI}{R}$ (3)

Since for a given value of W, the values of a, Y & I are constants. R is the constant so that the beam is bending uniformly into an arc of a circle of radius R.

CD = *l* and y is the elevation of the midpoint E of the beam so that y = EF

Then, from the property of the circle as shown in figure

EF x EG = CE x ED (4)



EF (2R – EF) = $(CE)^2$

($\because$ CE = ED, EG = 2R –EF, EF = y)

$Y(2R - y) = \left(\frac{l}{2}\right)^2 \left(\because CE = \left(\frac{l}{2}\right)\right)$

$2yR - y^2 = \frac{l^2}{4}$

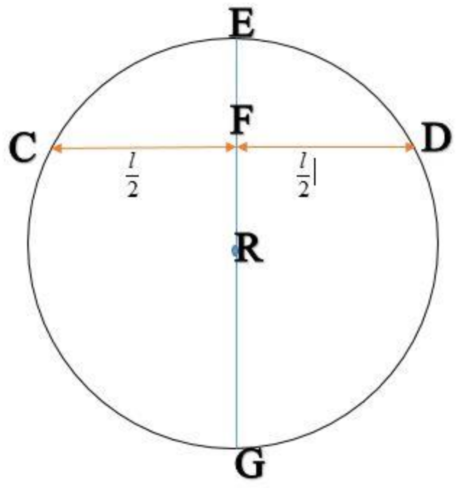
$y2R = \frac{l^2}{4} (\because y^2 \ is \ negligible)$

$y = \frac{l^2}{8R}$

$\frac{8y}{l^2} = \frac{1}{R}$

(or)

$\frac{1}{R} = \frac{8y}{l^2}$ (5)



From equations (3) & (5)

W a = $\frac{8y}{l^2}$ Y I

Y =$\frac{wl^2a}{8Iy}$

If the beam is of rectangular cross section, then I = $\frac{bd^3}{12}$

Where *b* is breadth and *d* the thickness of the beam.

If M is mass, the corresponding weight W = Mg,

Then, Y = $\frac{Mgl^2a}{8\frac{bd^3}{12}y}$

Hence, Y = $\frac{3Mgal^2}{2bd^3y}$

From which the young's modulus of the bar Y is determined

Experiment

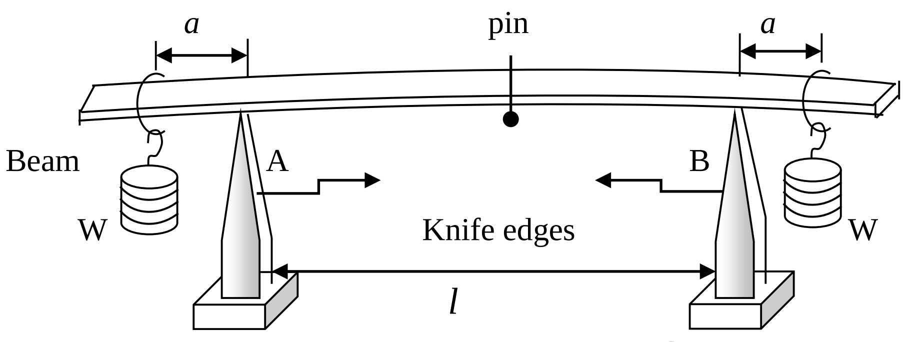
A rectangular beam (or bar) AB of uniform section is supported horizontally on two knife edges A and B as shown in fig. Two weight hangers of equal masses are suspended from the ends of the beam. A pin is arranged is vertically at the midpoint of the beam. A microscope is focused on the tip of the pin. Initial reading of the microscope in the vertical scale is noted.

Equal weights are added to both hangers simultaneously and the reading of the microscope in the vertical scale is noted. The experiment is repeated for decreasing order of magnitude of the equal masses. The observations are then tabulated and mean elevation (y) at the midpoint of the bar is determined

Sl.No	Load (M)	Microscopic readings			Depression (Y) for M=50 gms
		Loading	Unloading	Mean	
1	W				
2	W+50				
3	W+100				
4	W+150				
5	W+200				

The length of the bar between the knife edges '*l*' is measured. The distance of the one of the weight hangers from the nearest knife edge '*a*' is measured. The breadth (b) and thickness (d) of the bar are measured by using vernier callipers and screw gauge.

Young's modulus of the beam is determined by the relation $Y = \frac{3Mgal^2}{2bd^3y}$ Nm^{-2}



7. (i) Explain how young's modulus of non-uniform bending are determined both theoretically and experimentally?

Definition

If the beam is loaded at its mid-point, the depression produced does not form an arc of a circle. This type of bending is called non – uniform bending.

Description

Consider a uniform cross sectional beam **AB** of length ***l*** arranged horizontally on two knife edges near the ends **A** and **B**. A weight **W** is applied at the midpoint **O** of the beam. The reaction force at each knife edge is equal to $\frac{W}{2}$ in the upward direction. ***y*** is the depression at the midpoint **O**.

This bend beam is considered to be equivalent to two inverted cantilevers, fixed at **O** each of length $\frac{l}{2}$ and each loaded at knife edges $\mathbf{K_1}$ and $\mathbf{K_2}$ with a weight $\frac{W}{2}$. In case of cantilever of length ***l*** and load **W**, the depression is $y = \frac{Wl^3}{3Iy}$.

Hence, for a cantilever of length $\frac{l}{2}$ and load $\frac{W}{2}$ depression is

$$y = \frac{\left(\frac{W}{2}\right)\times\left(\frac{l}{2}\right)^3}{3IY} \rightarrow y = \frac{Wl^3}{48IY}$$

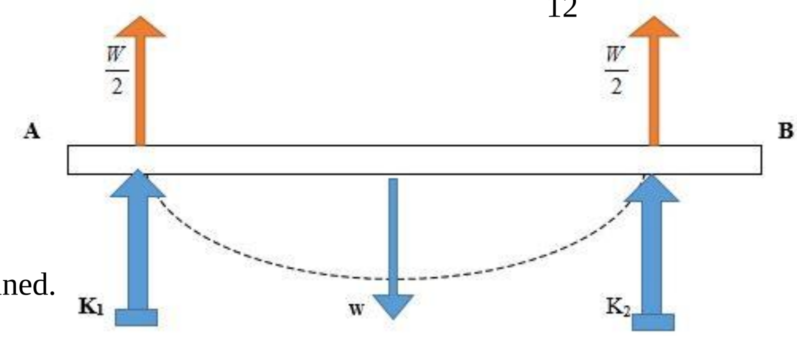
If **M** is the mass, the corresponding weight **W** is W = Mg

For a rectangular beam of breadth **b** and thickness **d**, the moment of inertia is $I = \frac{bd^3}{12}$

& hence, $y = \frac{Wl^3}{48\frac{bd^3Y}{12}}$

(or) $Y = \frac{Mgl^3}{4bd^3y}$ N/m^2.

From this the value of Y can be determined.



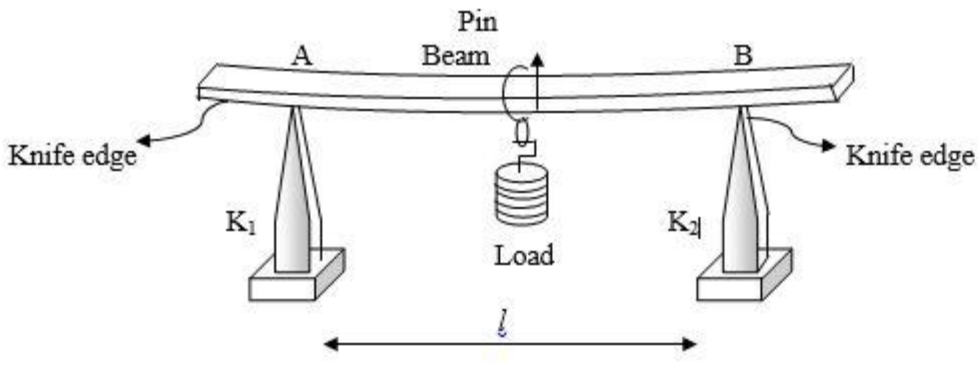
Experiment

The given beam **AB** of rectangular cross section is arranged horizontally on two knife edges $\mathbf{K_1}$ and $\mathbf{K_2}$ nears the ends A and B.. A weight hanger is suspended and a pin is fixed vertically at midpoint **O**. A microscope is focussed on the tip of the pin.
The initial reading on the vertical scale of the microscope is taken. A suitable mass **M** is added to the hanger. The beam is depressed. The cross wire is adjusted to coincide with the tip fo the pin. The microscopic reading is noted.
The depression corresponding to mass **M** is found. The experiment is repeated by increasing and decreasing the mass step by step. The corresponding readings are tabulated. The average value of depression ***y*** is found from the observation.

Sl.No	**Load (M)**	**Microscopic readings**			**Depression (Y) for M=50 gms**
		Loading	**Unloading**	**Mean**	
1	W				
2	W+50				
3	W+100				
4	W+150				
5	W+200				

The breadth ***b,*** thickness ***d*** and length ***l*** of the beam are determined.

Then the value of Young's modulus of the beam is $Y = \frac{Mgl^3}{4bd^3y}$



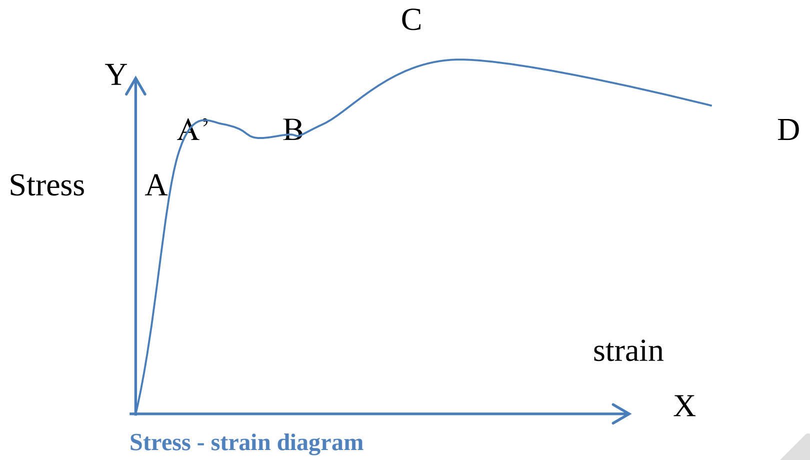
8. Explain stress strain diagram and three moduli of elasticity?

Stress – Strain diagram

Consider a wire rigidly fixed at one end and gradually loaded at the other end. The corresponding strain produced at each time is noted until the wire breaks down

A graph is plotted between strain along X – axis and stress along Y – axis is known as stress – strain diagram.

The following information regarding the behavior of solid materials are obtained by using this stress – strain diagram.



Stress - strain diagram

1. **Hooke's law**

The portion **OA** of the curve is a straight line. In this region, stress is directly proportional to strain. This means that up to **OA**, the material obeys Hooke's law. The wire is perfectly elastic. The point **A** is called the **limit of proportionality or proportional limit**

2. **Elastic limit**

The stress is further increased till a point **A'** is reached. This point **A'** lying near **A** denotes the **elastic limit.** Up to this point **A'**, the wire regains its original length if the stress is removed. If loaded beyond the elastic limit, the wire will not restore its original length.

3. **Yield point**

On further increasing the stress beyond the elastic limit, the curve bends and a point **B** is reached. In this region **A'B**, a slight increase in stress produces a larger strain in the material. The point **B** is called the **Yield point**. The value of the stress at the yield point is called yield strength of that material.

4. **Permanent set**

IN the region **A'B**, if stress is removed, the wire will never return to its original length but the wire is said to have taken a permanent set.

5. **Plastic range**

Beyond **B**, The extension (strain) increases rapidly without any increase in the load. This is known as **Plastic flow**

6. **Ultimate strength**

If the wire is further loaded, a point **C** is reached after which the wire begins to neck down or flow locally so that its cross sectional area no longer remains uniform. At this point **C**, the wire begins to thin down at some point where it finally breaks. At this point **C**, the value of the developed stress is maximum and is called **ultimate tensile strength** of the given material

7. Breaking point

The point **D** is known as the breaking point, where the wire breaks down completely. The stress corresponding to **D** is called breaking stress.

Uses of stress – strain diagram

1. It is used to determine the elastic strength, yield strength and tensile strength of metals
2. It is used to estimate the working stress and safety factor of an engineering material. The lower value of the safety factor are adopted to keep the structure for long life.
3. The area under the curve in elastic region gives the energy required to deform elastically. Whereas the area under the curve in ultimate tensile strength gives the energy required to deform plastically.
4. It is also used to identify the ductile and brittle materials

Types of moduli of elasticity

There are three types of modulus of elasticity namely Young's modulus, Rigidity modulus and bulk modulus

(i) Young's modulus

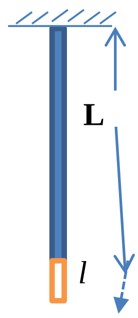
Within the elastic limit, the ratio of longitudinal stress to longitudinal strain is called young's modulus of elasticity **(E)**

Young's modulus of elasticity (E) = $\frac{\textbf{Longitudinal Stress}}{\textbf{Longitudinal Strain}}$

The longitudinal force F is applied normally to a cross sectional area '**a**' of a wire as shown in fig 1.2.

Longitudinal Stress = $\frac{\textbf{Longitudinal force (F)}}{\textbf{Area (a)}}$

If **L** is the original length and '***l***' is the change in length due to the applied force, then



Longitudinal Stress = $\frac{\textbf{change in length } (l)}{\textbf{Original length (L)}}$Wire

∴Young's modulus of elasticity (E) = $\frac{\textbf{Longitudinal Stress}}{\textbf{Longitudinal Strain}} = \frac{F/a}{l/L} = \frac{FL}{al}$ N/m^2 Fig 1.2

(ii) Rigidity modulus (n)

Within the elastic limit, the ratio of the shearing (tangential) stress to shearing strain is called rigidity modulus.

Rigidity modulus (n) = $\frac{\textbf{Shearing (Tangential) Stress}}{\textbf{Shearing Strain}}$

Consider a rectangular block fixed at its lower face EFGH. A force **F** is applied tangentially on its upper face ABCD as shown in fig 1.3.

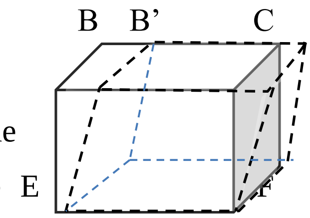


Fig 1.3 Rigidity modulus

C'

A force of reaction of the same magnitude **F** acts on the lower face EFGH in the opposite direction. These two equal and opposite forces constitute a couple. Due to this couple, the body gets deformed and its shape changes. All the four vertical sides are rotated (sheared)through an angle θ. This angle θ is known as the shearing strain

φ

Shearing Stress = $\frac{\textbf{Tangential force (F)}}{\textbf{Area of the face ABCD (a)}}$

Shearing strain (φ) = tan φ = $\frac{AA'}{AF} = \frac{l}{L}$

∴Rigidity modulus (n) = $\frac{\textbf{Shearing (Tangential) Stress}}{\textbf{Shearing Strain}} = \frac{F}{a\varphi} = \frac{FL}{al}$ N/m^2

(iii) **Bulk Modulus**

Within the elastic limit of a body, the ratio of the volume stress to the volume strain is called bulk modulus of elasticity

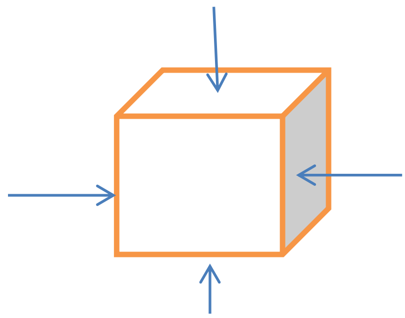
Bulk modulus (K) = $\frac{\textbf{Volume Stress}}{\textbf{Volume Strain}}$

When deforming force **F** acts normally on all the faces of a solid body, the body undergoes a change in its volume but not in the shape

Volume of the body = **V**

Surface area of each face subjected to the force = **A**

Change in volume = d*v*



Volume stress = $\frac{\textbf{Normal force (F)}}{\textbf{Area (A)}}$= Pressure (**P**)

Volume strain = $\frac{\textbf{change in volume } (dv)}{\textbf{Original volume (V)}}$

∴Bulk modulus (K) = $\frac{\textbf{Volume Stress}}{\textbf{Volume Strain}} = \frac{-P}{\frac{dv}{V}} = \frac{-PV}{dv}$ N/m^2

9. Write short notes on (i) I – Shape girders (iii) factors affecting elasticity and tensile stress

I – Shape Girders

"The girders with upper and lower section broadened and the middle section tapered, so that it can withstand heavy loads over it "

Loaded bearing beam is called girder

Explanation:

In general, any girder supported at its two ends as on the opposite walls of a room, bends under its own weight and a small depression is produced at the middle portion. This may also be caused when loads are applied to the beams

Due to depression produced, the upper parts of the girder above the neutral axis contracts, while the lower parts below the neutral axis expands. i.e., stress have maximum value at the top and bottom. The stress progressively decreases as it approaches towards neutral axis. Therefore upper and lower surfaces of the girder must be stronger than the intervening part in form of I(I – shape girders)

Minimization of Depression

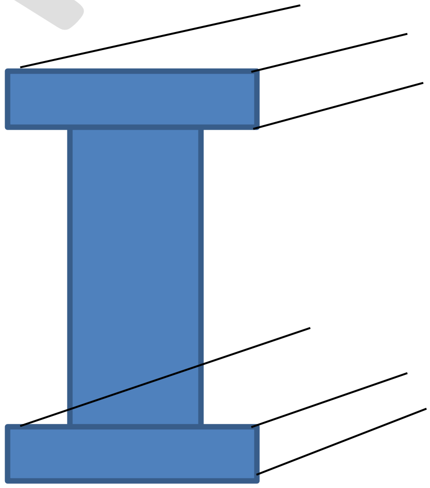
Since the length is fixed quantity, the breadth and thickness may be adjusted by making the girder of large depth and small breadth (i.e., upper and lower part is made broader than the center part). Hence the volume of girder is increased and the depression produced is reduced. In the case of rectangular cross section, $y = \frac{4Wl^3}{Ybd^3}$

Advantages

(i) More stable
(ii) High durability

Applications

(1) They are used in construction of bridges
(2) They are used in production of iron rails employed over railway tracks
(3) They are used as supporting beams for ceilings



Factors affecting Elasticity and Tensile strength

1. **Effect of stress**
   The action of large constant stress or repeated number of cycles of stresses acting on a body decreases the elasticity of the body gradually
2. **Effect of change in temperature**
   A change in the temperature affects the elastic properties of a material. A rise in temperature usually decreases the elasticity of the material. A carbon filament which is highly elastic at normal temperature becomes plastic when it is at high temperature. Similarly, a decrease in temperature will increase the elastic property. Lead is not a very good elastic material. But at low temperature, it becomes a very good elastic material. However, in some cases like the invar steel, the elasticity is not affected by any change in temperature
3. **Effect of impurities**
   The elastic property of a material is either increased or decreased due to the addition of impurities. It depends up on the elastic or plastic properties of the impurities added. If carbon is added with minute quantities of molten iron, the elastic properties of iron are increased enormously. If more carbon is added, its elastic properties are decreased. Similarly the addition of potassium or copper in gold increases the elastic properties of gold.
4. **Effect of hammering, rolling and annealing**
   Operations like hammering and rolling help in breaking up the crystal grains into smaller units and results in an increase of their elastic properties. Operations like annealing help in forming larger crystal grains. Hence there is a decrease in their elastic properties or an increase in their softness or plasticity of the material
5. **Effect of crystalline nature**
   For a given metal, the modulus of elasticity is more when it is in single crystal form and in the polycrystalline state, its modulus of elasticity is comparatively small.