4. QUANTUM PHYSICS

Part – A

1. **Explain Planck's hypothesis? (or) What are the postulates of Planck's quantum theory? (or) what are the assumptions of quantum theory of black body radiation?**

   (i) The electrons in the black body are assumed as simple harmonic oscillator

   (ii) The frequency of radiation emitted by an oscillator is the same as that of the frequency of vibration

   (iii) The oscillators (electrons) radiate energy in a discrete manner and not in a continuous manner.

   (iv) The oscillators exchanges energy in the form of either absorption or emission within the surroundings in terms of quanta of magnitude 'hν'

2. **What is Compton Wavelength?**

   The shift in wavelength corresponding to the scattering angle of 90° is called Compton wavelength.

   W.K.T Compton shift (Δλ) = $\frac{h}{m_0C}(1 - \cos 90°) = \frac{6.626 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^{-8}} = 0.02424A°$

3. **State De-Broglie's Hypothesis (or) explain the concept of wave nature? (or) What is meant by matter waves? Give the origin of concept?**

   The light exhibit dual nature such as a particle & wave. De-Broglie suggested that an electron, which is a particle, can also behave as a wave and exhibit the dual nature. Thus the wave associated with the material particle are called matter waves $\therefore De - Broglie\ wavelength\ (\lambda) = \frac{h}{mv}$

4. **Give the importance of Planck's radiation formula?**

   (i) it explains all regions of black body spectrum

   (ii) it is based on quantum theory

   (iii) it is used to derive other laws related to black body radiation

5. **What is the physical significance of a wave function?**

   (i) The probability of finding the particle in space at any given instant of time is characterized by a function ψ (x,y,z) called wave function

   (ii) It relates the particle and the wave statistically

   (iii) It gives the information about the particle behavior

   (iv) It is a complex quantity

   (v) $\psi\psi^*$ is a probability density of the particle , which is real and positive

6. **What is black body and what are its characteristics?**

   A perfect black body is said to be a perfect absorber, since it absorbs all the wavelength of the incident radiation. The black body is a perfect radiator, because its radiates all the wavelengths absorbed by it. This phenomenon is called black body radiation.

7. **Define Stefan – Boltzmann's law?**

   It states that "the total amount of energy radiated per second per unit area of a perfect black body is directly proportional to fourth power of the absolute tempecture "

   i.e., $E \propto T^4$ or $E = \sigma T^4$

   Where σ is a Stefan's constant and $\sigma = \frac{2\pi^5 K^4}{15\, h^3 C^2}$ = 5.67 x 10^{-8} $Wm^{-2}K^{-4}$

8. **State Kirchoff's law of radiation?**

   Ratio of emissive power to the coefficient of absorption of any given wavelength is the same for all bodies at a given temperature and is equal to the emissive power of the black body at that temperature

$$\frac{e_\lambda}{a_\lambda} = E$$

9. **Define Wien's Displacement law? Give its importance?**

It states that "the wavelength corresponding to the maximum wavelength is inversely proportional to the absolute tempecture" i.e., λ_{max} T = Constant

Limitations: it holds good only at shorter wavelengths.

10. **Define Rayleigh's – jeans Law? Give its limitations?**

It states that "the energy distribution is directly to the absolute tempecture and inversely proportional to the fourth power of the wavelength

i.e., $E_\lambda = \frac{8\pi KT}{\lambda^4}$

This law holds good only for longer wavelength regions.

11. **What is meant by photon? Give any two properties?**

Photons are discrete energy values in the form of small quanta's of definite frequency (or) wavelength.

Properties:

(i) They does not have any charge and they will not intense

(ii) The energy & momentum of the photon is given by E = hυ and p = mc

12. **Define Compton effect and Compton shift?**

When a photon of energy "hυ" collides with a scattering element. The scattering beam has two components as one of them have same frequency (or) wavelength as that of incident radiation and the other have lower frequency (or) higher wavelength . This effect is called Compton effect. The shift in wavelength due to scattered x- rays is called Compton shift.

13. **Define Eigen value and Eigen function?**

Energy of a particle moving in one dimensional box of width 'a' is $E_n = \frac{n^2h^2}{8ma^2}$.for each value of 'n' there is a energy level. Where E_n is called Eigen value.

For every quantum state, there is a corresponding wave function 'ψ_n' called Eigen function given by $\psi_n = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$

14. **What is meant by degenerate (or) non – degenerate state?**

For various combinations of quantum numbers if we get some Eigen value at different eigen functions, then it is called degenerate state.

For various combinations of quantum numbers if we get same eigen values & eigen functions, then it is called non degenerate state.

15. **What are the properties of matter waves?**

(i) Matter waves are not electromagnetic waves.

(ii) Matter waves are new kind of waves in which due to the motion of the charged particles, electromagnetic waves are produced.

(iii) Lighter particles will have high wavelength

(iv) Particles moving with less velocity will have high wavelength

(v) The velocity of matter wave is not a constant, it depends on the velocity of the particle.

(vi) If the velocity of the particle is infinite then the wavelength of matter wave is indeterminate(λ=0)

(vii) The wave and particle aspects cannot appear together

(viii) Locating the exact position of the particle in the wave is uncertain

16. Distinguish any four differences between TEM and STEM?

S.NO	TEM	STEM
1	Magnification is 10,00,000 times	Magnification is more than 10,00,000 times
2	Resolution is 0.2nm	Resolution is 0.1nm
3	Resultant image is a 2 – D image	Resultant image is a 3 – D image
4	Cost is low	Cost is high

17. For a free particle moving with a one dimensional potential box, the ground state energy cannot be zero, why?

For a free particle moving within a one dimensional potential box, when $n=0$ the wave function is zero for all values of x i.e., it is zero even within the potential box. This would mean that the particle is not present within the box. Therefore the state with $n=0$ is not allowed.

18. What is meant by tunneling effect?

In quantum mechanics, a particle having lesser energy (E) than the barrier potential (V) can easily cross over the potential barrier having a finite width *'a'* even without climbing over the barrier by tunneling through the barrier. This process is called Tunneling.

19. Mention the occurrence of tunneling effect?

(i) Josephson effect – electron pairs in the super conductor's tunnel through the barrier layer giving rise to Josephson current

(ii) Emission of alpha particles by radioactive nucleui

(iii) Tunneling diodes

(iv) Electron tunnels through insulating layer act as a switch by tunneling effect.

20. What is meant by energy spectrum of a black body? what do you infer from it?

The distribution of energy for various wavelength at various temperatures is known as energy spectrum of a black body

Inference

(i) The energy distribution is not uniform at any given temperature

(ii) When temperature increases, energy decreases

(iii) The total energy emitted at any particular temperature can be found by the area traced by the curve

Part - B

1. Derive an expression for Planck's radiation law from the average energy emitted by a black body & deduce Planck's formula to prove the Wien's displacement law, Rayleigh- Jeans law & Stefan's – Boltzmann law?

Assumptions: Planck derived an expression for the energy distribution, with the following assumptions:

(i) A black body radiator contains electrons or so called simple harmonic oscillators, which are capable of vibrating with all possible frequencies.

(ii) The frequency of radiation emitted by an oscillator is the same as that of the frequency of vibrating particles

(iii) The oscillators (electrons) radiate energy in a discrete manner and not in a continuous manner.

(iv) The oscillators exchanges energy in the form of either absorption or emission within the surroundings in terms of quanta of magnitude 'hν' as in fig. 4.1

(v) The vibrating particles can radiate energy when the oscillator moves from one state to another

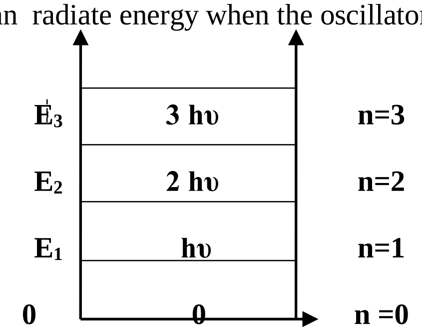


Fig 4.1

i.e., E = nhν

where n = 0,1,2,3…..

Thus the exchange of energy are limited to a discrete set of values say 0,hν,2hν,3hν ……. For 0, E, 2E, 3E,… for *n* number of oscillators.

Planck's Radiation Law

Let us consider 'N' number of oscillators with their total energy as E_r.

∴The average energy of an oscillator is given by $E = \frac{E_T}{N}$ (1)

If N_0, N_1, N_2, N_3, ….. are the oscillators of energy 0,E,2E,3E,….respectively then we can write

(i) The total number of oscillators $N = N_0 + N_1 + N_2 + N_3 + …….$ (2)

(ii) Total energy of oscillators $E_T = 0N_0 + EN_1 + 2EN_2 + 3EN_3 + ……..$ (3)

According to Maxwell's distribution formula, the number of particles in the oscillatory system having an energy is given by $N = N_0 e^{-E/KT}$ (4)

Where K - Boltzmann constant; T – Temperature

For various values of oscillators, i.e., n = 0, 1, 2, 3…... the number of oscillators N_0, N_1, N_2, N_3, ….. as:

(i) For n=0 : $N_0 = N_0 e^0$

(ii) For n=1 : $N_1 = N_0 e^{-1E/KT}$

(iii) For n=2 : $N_2 = N_0 e^{-2E/KT}$

(iv) For n=3 : $N_3 = N_0 e^{-3E/KT}$

Therefore the total number of oscillators can be obtained by substituting the values of N_0, N_1, N_2, N_3, ….. in equation (2), thus

$N = N_0 e^0 + N_0 e^{-1E/K_BT} + N_0 e^{-2E/KT} + N_0 e^{-3E/KT} + ………$

$N = N_0 [1+ e^{-1E/KT} + e^{-2E/KT} + e^{-3E/KT} \ldots\ldots]$

Put $x = e^{-E/KT}$, then equation (5) becomes, $N = N_0[1+ x + x^2 + x^3 + \ldots\ldots \ldots]$ (5)

We know, $1 + x + x^2 + x^3 + \cdots .. = \frac{1}{(1-x)}$. Therefore we can write equation (5) as

The total number of oscillators $N = N_0 \frac{1}{e^{\frac{-E}{kT}}}$ (6)

Similarly by substituting the values of N_0, N_1, N_2, N_3, ….. in equation (3), the total energy can be written as

$E_T = 0N_0e^0 + EN_0e^{-1E/KT} + 2EN_0e^{-2E/KT} + 3EN_0e^{-3E/KT} + \ldots\ldots\ldots$

$E_T = N_0Ee^{-E/KT} [1+2e^{-E/KT}+3e^{-2E/KT}+\ldots\ldots]$ (7)

Put $x = e^{-E/KT}$, then equation (7) becomes $E_T = N_0Ee^{-E/KT} [1+2x +3x^2 + 4x^3 +\ldots\ldots]$

We know $1 + 2x + 3x^2 + \cdots = \frac{1}{(1-x)^2}$

Therefore equation (7) becomes

The total energy of oscillators $E_T = N_0 e^{\frac{-E}{kT}} \left[\frac{1}{\left(1-e^{-E/kT}\right)^2}\right]$ (8)

Now the average energy from equation (i) is

Average energy $\overline{E} = \frac{E_T}{N} = \frac{N_0 E e^{\frac{-E}{kT}}}{N_0\left[\frac{1}{1-e^{\frac{-E}{kT}}}\right]} \frac{1}{\left[1-e^{\frac{-E}{kT}}\right]^2}$ (9)

Or

$$\overline{E} = \frac{Ee^{\frac{-E}{kT}}}{1-e^{\frac{-E}{kT}}} \text{ (or) } \overline{E} = \frac{E}{\left(\frac{1}{e^{\frac{-E}{kT}}} - \frac{e^{\frac{-E}{kT}}}{e^{\frac{-E}{kT}}}\right)} \text{ (or) } \overline{E} = \frac{E}{e^{\frac{E}{kT}}-1} \quad (10)$$

Substituting the values of E= hν in the above equation, we get

$$\overline{E} = \frac{h\nu}{e^{h\nu/K_BT}-1}$$

Equation (10) represents the average energy of the oscillator.

The number of oscillators per unit volume within the range of frequency υ and υ + d υ is given by

$N = \frac{8\pi\nu^2}{c^3} d\nu$ (11)

Energy density $E_\upsilon\, d\upsilon$ (or) Total energy per unit volume = No. of oscillators per unit volume **x** Average energy of an oscillator

$$\boldsymbol{E_\nu d\nu = N.\overline{E}} \qquad (12)$$

Substituting equations (10) and (11) in equation (12), we get $E_\nu d\nu = \frac{8\pi\nu^2}{c^3} d\nu \frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$

(or) $$E_\nu d\nu = \frac{8\pi h\nu^3}{c^3(e^{\frac{h\nu}{kT}}-1)} d\nu \qquad (13)$$

Planck's radiation law in terms of wavelength.

We know $\nu = \frac{c}{\lambda}$

Differentiating numerically we get, $|d\nu| = \frac{c}{\lambda^2} d\lambda$

Substituting the value of υ and d υ, we get $E_\lambda d\lambda = \frac{8\pi hc^3}{c^3\lambda^3} \frac{c}{\lambda^2} d\lambda \frac{1}{\left(e^{hc/\lambda KT}-1\right)}$

(or) $$E_\lambda = \frac{8\pi hc}{\lambda^5\left(e^{hc/\lambda KT}-1\right)} \qquad (14)$$

This law has good agreement with all the experimental results. It also helps to derive the Stefan-Boltzmann law, Wien's displacement law and Rayleigh Jean's law.

Wien's Displacement law:

It holds good only at shorter wavelengths, hence λ<<1, 1 / λ>>1. Therefore $\left[e^{hc/\lambda KT}-1\right] \approx e^{hc/\lambda KT}$

Hence equation (14) becomes,

$$E_\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{\left[e^{hc/\lambda KT}\right]}\right] = C_1\lambda^{-5}e^{-\frac{C_2}{\lambda T}}$$

Where $C_1 = 8\pi hc$ & $C_2 = hc / k$

This is the empirical formula for Wien's displacement law

Rayleigh – Jean's Law:

It holds good only for longer wavelength, hence hence λ>>1, 1 / λ<<1.

Therefore $e^{hc/\lambda KT}$ = 1 + hc / λKT+ ½ (hc / λKT)^2 + ………Since the higher order terms is very small , the terms having powers are neglected. Hence, $e^{hc/\lambda K_B T}$ = 1 + hc / λKT

$$\therefore E_\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{\left[1+\frac{hc}{\lambda KT}-1\right]}\right] = \frac{8\pi KT}{\lambda^4}$$

2. Give the theory of Compton effect? Explain briefly about its experimental verification?

Compton Effect: When a beam of monochromatic radiation such as X-rays, γ rays etc., of high frequency is allowed to fall on a fine scatterer, the beam is scattered into two components viz,

(i) One component having the same frequency (or) wavelength as that of the incident radiation so called **unmodified radiation,** and

(ii) The other component having lower frequency (or) higher wavelength compared to incident radiation, so called **modified radiation**.

This effect of scattering is called **Compton Effect and the change in wavelength of scattered X – rays is known as Compton shift.**

Thus as a result of Compton scattering, we get (i) Unmodified radiation (ii) Modified radiation and (iii) a recoil electron.

Theory of Compton Shift

Principle : In Compton scattering the collision between a photon and an electron is considered. Then by applying the laws of conservation of energy and momentum, the expression for Compton wavelength is derived.

Assumptions

1. The collision occurs between the photon and an electron in the scattering material.
2. The electron is free and is at rest before collision with the incident photon.

Now, let us consider a photon of energy 'hν' colliding with an electron at rest of mass m_0.

During the collision process, a part of energy is given to the electron, which in turn increases the kinetic energy of the electron and hence it recoils at an angle of Φ with mass 'm' and velocity 'v' as in fig. 4.10. The scattered photon moves with an energy $h\nu'$ with longer wavelength than hν, at an angle θ with respect to the original direction.

Let us find the energy and momentum components before and after collision process.

Energy before collision

(i) Energy of the incident photon = hν

(ii) Energy of the electron at rest $= m_0c^2$

Where m_0 is the rest mass energy of the electron.

Total Energy before Collision $= h\nu + m_0c^2$ (1)

Energy after collision

(i) Energy of the scattered photon = $h\nu'$

(ii) Energy of the recoil electron = mc^2

Where m is the mass of the electron moving with velocity 'v'

Total energy after collision = $h\nu' + mc^2$ (2)

We know according to the law of conservation of energy,

Total energy before collision = Total energy after collision

Therefore Equation (1) = Equation (2)

(i.e.,) $h\nu + m_0c^2 = h\nu' + mc^2$ (3)

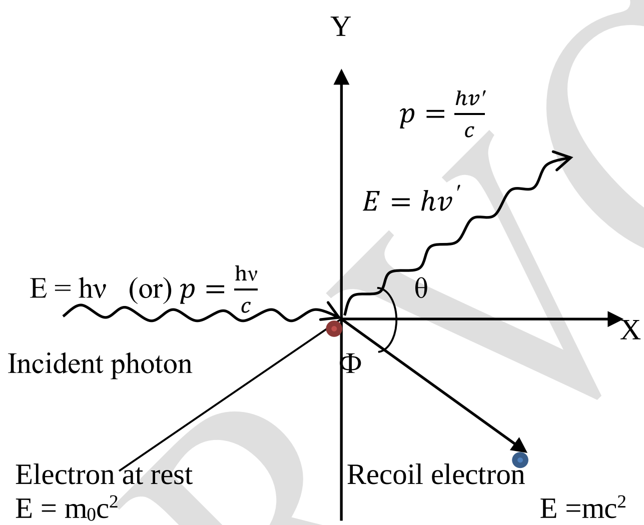


Fig 4.10

X-Component of Momentum before Collision

(i) X-component momentum of the incident photon = $h\nu/c$

(ii) X-component momentum of the electron at rest = 0

Total X-Component of momentum before collision = $h\nu/c$ (4)

X-Component of Momentum after Collision

(i) X-component momentum of the scattered photon can be calculated from fig. 4.11

In ΔOAB $\cos\theta = M_x / (h\nu'/c)$

X-component momentum of the scattered photon (M_x) = $h\nu'/c \cos\theta$

(ii) X-component momentum of the recoil electron can be calculated from fig. 4.11

In ΔOBC $\cos \Phi = M_x / mv$

X-component momentum of the recoil electron (M_x) = $mv \cos \Phi$

Total X-component of momentum after collision = $h\nu'/c \cos \theta + mv \cos \Phi$ (5)

We know according to the law of conservation of momentum,

Total momentum before collision = Total momentum after collision

i.e., Equation (4) = Equation (5)

$$\frac{h\upsilon}{c} = \frac{h\upsilon'}{c}\cos\theta + mv\cos\phi \quad (6)$$

Y-Component of Momentum before Collision

(i) Y-component momentum of the incident photon = 0

(ii) Y-component momentum of the electron at rest = 0

Total Y-Component of momentum before collision = 0 (7)

Y-Component of Momentum after Collision

(i) Y-component momentum of the scattered photon can be calculated from fig. 4.11.

In ΔOAE $\sin \theta = M_y / (h\nu'/c)$

Y-component momentum of the scattered photon= $\frac{h\upsilon'}{c}\sin\theta$

(ii) Y-component momentum of the recoil electron can be calculated from fig. 4.11

In ΔOCD $\sin \Phi = -M_y / mv$

Y-component momentum of the recoil electron = $-mv \sin \Phi$

Total Y-component of momentum after collision = $\frac{h\upsilon'}{c}\sin\theta - mv\sin\phi$ (8)

According to the law of conservation of momentum,

Total momentum before collision = Total momentum after collision

i.e., Equation (7) = Equation (8)

$$0 = = \frac{h\nu'}{c}\sin \theta - mv \sin \Phi \quad (9)$$

from equation (6), we can write $\frac{h\upsilon}{c} - \frac{h\upsilon'}{c}\cos\theta = mv\cos\phi$

(or) $h(\nu - \nu' \cos\theta) = mcv \cos\Phi$ (10)

from equation (9) we can write

$h\nu' \sin\theta = mcv \sin\Phi$ (11)

squaring and adding equation (10) & (11)

$h^2(\nu^2 - 2\nu\nu'\cos\theta + (\nu')^2\cos^2\theta) + h^2(\nu')^2\sin^2\theta = m^2c^2v^2(\cos^2\Phi + \sin^2\Phi)$

since $\cos^2\Phi + \sin^2\Phi = 1$ and $h^2(\nu')^2[\cos^2\theta + \sin^2\theta] = h^2(\nu')^2$ we get

$h^2(\nu^2 - 2\nu\nu'\cos\theta + (\nu')^2) = m^2c^2v^2$

(or) $h^2\nu^2 - 2h^2\nu\nu'\cos\theta + h^2(\nu')^2 = m^2c^2v^2$ (12)

from equation (3), we can write $mc^2 = m_0c^2 + h(\nu - \nu')$

Squaring on both sides we get

$m_0^2c^4 + 2h\, m_0c^2(\nu - \nu') + h^2[\nu^2 - 2\nu\nu' + (\nu')^2] = m^2c^4$

$m_0^2c^4 + 2hm_0c^2(\nu - \nu') + h^2\nu^2 - 2h^2\nu\nu' + h^2(\nu')^2 = m^2c^4$ (13)

Subtracting equation (12) from equation (13) we get

$m_0^2c^4 + 2h\, m_0c^2(\nu - \nu') - 2h^2\nu\nu'(1 - \cos\theta) = m^2c^2(c^2 - v^2)$ (14)

From the theory of relativity, the relativistic formula for the variation of mass with velocity of the electron is given by $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

Squaring, we get $m^2 = \frac{m_0^2}{\frac{c^2 - v^2}{c^2}}$ (or) $m^2 = \frac{m_0^2c^2}{c^2 - v^2}$

$m^2(c^2 - v^2) = m_0^2c^2$ (15)

Now let us multiply c^2 on both sides of this equation to make it similar to equation (14)

$m^2c^2(c^2 - v^2) = m_0^2c^4$ (16)

Now let us equate equations (16) and (14)

$$m_0^2c^4 = m_0^2c^4 + 2h\, m_0c^2 (\nu - \nu') - 2h^2\nu\nu'(1-\cos\theta)$$

$$2h\, m_0c^2 (\nu - \nu') = 2h^2 \nu\nu'(1-\cos\theta)$$

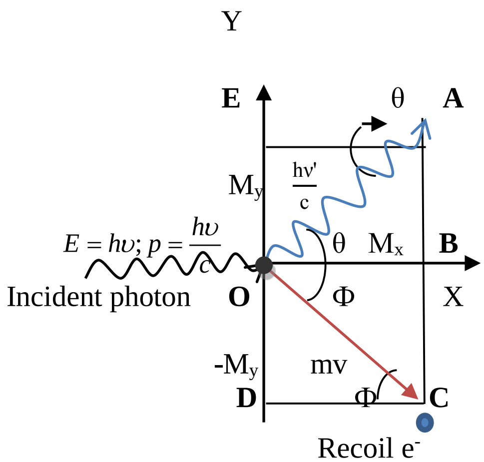


Fig. 4.11

(or) $$\frac{\upsilon-\upsilon'}{\upsilon\upsilon'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

(or) $$\frac{\upsilon}{\upsilon\upsilon'} - \frac{\upsilon'}{\upsilon\upsilon'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

(or) $$\frac{c}{\upsilon'} - \frac{c}{\upsilon} = \frac{h}{m_0c}(1-\cos\theta)$$

(or) $$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta) \Rightarrow \Delta\lambda = \frac{h}{m_0c}(1-\cos\theta) \qquad (18)$$

Equation (18) represents the shift in wavelength, i.e. Compton shift which is independent of the incident radiation as well as the nature of the scattering substance.

Thus the shift in wavelength or Compton shift purely depends on the angle of scattering.

Special cases

Case (i) when θ = 0; cos θ = 1

Equation (18) becomes $\Delta\lambda = 0$ .This implies that at $\theta = 0$ the scattering is absent and the out coming radiation has the same wavelength or frequency as that of the incident radiation. Thus the output will be a single peak as shown in figure 4.12 (a).

Case (i) when $\theta = 90^0$; cos θ = 0

Equation (18) becomes $\lambda = \frac{h}{m_oC}$ Substituting *h, mₒ and C , Δλ = 0.02424A°*

This wavelength is called COMPTON WAVELENGTH, which has a good agreement with the experimental results as shown in fig.4.12(c)

Case (i) when θ = 180^0; cos θ = -1

Equation (18) becomes $\lambda = \frac{h}{m_o C}[1-(-1)] = \frac{2h}{m_o C}$

Substituting *h*, m_o *and C*, $\Delta\lambda = 0.04848A^o$

Thus for θ = 180^0 the shift in wavelength is found to be maximum as shown in fig 4.12(d).

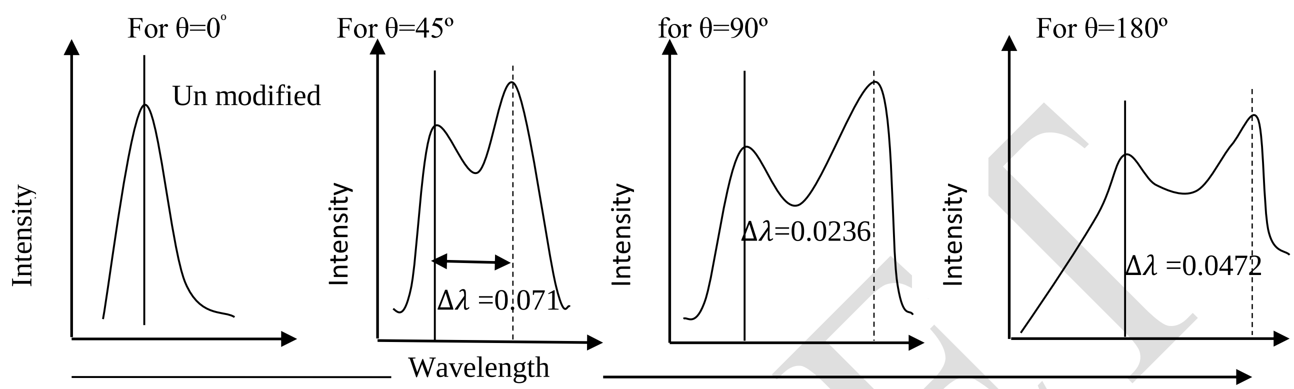


Fig 4.12 (a) Fig 4.12 (b) Fig 4.12(c) Fig 4.12(d)

EXPERIMENTAL VERIFICATION OF COMPTON EFFECT:

Principle

When a photon of energy 'hν' collides with a scattering element, the scattered beam has two components, viz, one of the same frequency (or) wavelength as that of the incident radiation and the other has lower frequency (or) higher wavelength compared to incident frequency (or) wavelength. This effect is called Compton effect and the shift n wavelength is called Compton shift.

Construction

It consists of an X-ray tube for producing X-rays. A small block of carbon C (scattering element) is mounted on a circular table as in fig. 4.13

A Bragg's spectrometer (B_s) is allowed to freely swing in an arc about the scattering element to catch the scattered photons. Slits S_1 and S_2 helps to focus the X-rays onto the scattering element.

Working

X-rays of monochromatic wavelength 'λ' is produced from an X-ray tube and is made to pass through the slits S_1 and S_2. These X-rays are made to fall on the scattering element. The scattered X-rays are received with the help of the Bragg's spectrometer and the scattered wavelength is measured .Now an ionization energy is replaced at the target to measure the intensity for the corresponding wavelength.

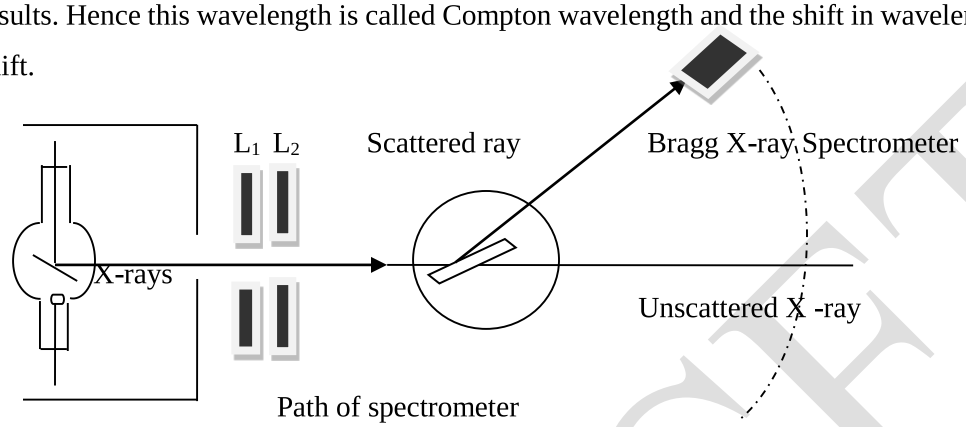
The experiment is repeated for various scattering angles and the scattered wavelengths and the corresponding intensities are measured. The experimental results are plotted as in fig. 4.13

In this fig. when the scattering angle θ = 0^0, the scattered radiation peak will be the same as that of the incident radiation peak 'A'. Now when the scattering angle is increased, for one incident radiation peak

A of wavelength (λ) we get two scattered peaks A and B. Here the peak 'A' is found to be of same wavelength as that of the incident wavelength and the peak B is of greater wavelength than the incident radiation.

The shift in wavelength (or) difference in wavelength (Δλ) of the two scattered beams is found to increase with respect to the increase in scattering angle.

At $\theta = 90^0$, Δλ is found to be 0.0236 ≈ 0.02424, which has good agreement with the theoretical results. Hence this wavelength is called Compton wavelength and the shift in wavelength is called Compton shift.



3.Derive Schröedinger's Time dependent & time independent wave equations? Give the physical significance of ψ?

Schröedinger Time Dependent wave function

A particle can behave as a wave only under motion. So, it must be accelerated by a potential field

∴, Total energy (E) = Potential Energy (V) + Kinetic Energy

i.e., $E = V + \frac{1}{2}mv^2$

(or) $E = V + \frac{1}{2}\frac{m^2v^2}{m}$

(or) $E = V + \frac{p^2}{2m}$ [Since p = mv]

(or) $E\Psi = V\Psi + \frac{p^2}{2m}\Psi$ (1)

According to classical mechanics if 'x' is the position of the particle moving with the velocity 'v', then the displacement of the particle at any time 't' is given by

$y = Ae^{-i\omega\left(t - \left(\frac{x}{v}\right)\right)}$

Where ω is the angular frequency of the particle

Similarly in quantum mechanics the wave equation Ψ (x, y, z, t) represents the position (x, y, z) of a moving particle at any time't' and is given by

$$\Psi(x,y,z,t) = Ae^{-i\omega\left(t-\left(\frac{x}{v}\right)\right)} \qquad (2)$$

We know that angular frequency $\omega = 2\pi\nu$

∴ Equation (2) becomes

$$\Psi(x,y,z,t) = Ae^{-i2\pi\left(\upsilon t-\left(\frac{x\upsilon}{v}\right)\right)} \qquad (3)$$

We know $E = h\upsilon$ (or) $\upsilon = \frac{E}{h}$ (4)

Also, if 'v' is the velocity of the particle behaving as a wave,

Then the frequency $\upsilon = \frac{v}{\lambda}$ (or) $\frac{v}{\upsilon} = \frac{1}{\lambda}$ (5)

Substituting equations (4) & (5) in equation (3), we get

$$\Psi(x,y,z,t) = Ae^{-i2\pi\left(\left(\frac{Et}{h}\right)-\left(\frac{x}{\lambda}\right)\right)} \qquad (6)$$

If 'p' is the momentum of the particle, then the de-Broglie wavelength

is given by $\lambda = \frac{h}{mv} = \frac{h}{p}$ (7)

Substituting equation (7) in (6) we get

$$\Psi(x,y,z,t) = Ae^{-i2\pi\left(\left(\frac{Et}{h}\right)-\left(\frac{px}{h}\right)\right)}$$

(or) $\Psi(x,y,z,t) = Ae^{-i\frac{2\pi}{h}(Et-px)}$

Since $\hbar = \frac{h}{2\pi}$ we can write $\Psi(x,y,z,t) = Ae^{-\frac{i}{\hbar}(Et-px)}$ (8)

Differentiating equation (8) partially with respect to 'x' we get

$$\frac{\partial\Psi}{\partial x} = Ae^{-\frac{i}{\hbar}(Et-Px)}\left(\frac{ip}{\hbar}\right)$$

Differentiating once again partially with respect to 'x' we get

$$\frac{\partial^2\Psi}{\partial x^2} = Ae^{-\frac{i}{\hbar}(Et-Px)}\left(\frac{i^2p^2}{\hbar^2}\right)$$

Since $\Psi(x,y,z,t) = Ae^{-\frac{i}{\hbar}(Et-px)}$ and $i^2 = -1$, we can write

$$\frac{\partial^2\Psi}{\partial x^2} = \Psi(x,y,z,t)\left(\frac{-p^2}{\hbar^2}\right)$$

(or) $p^2\Psi = -\hbar^2 \dfrac{\partial^2\Psi}{\partial x^2}$ (9)

Differentiating equation (8) partially with respect to‘t’ we get

$$\frac{\partial\Psi}{\partial t} = Ae^{-\frac{i}{\hbar}(Et-Px)}\left(\frac{-iE}{\hbar}\right)$$

(or) $\dfrac{\hbar}{-i}\dfrac{\partial\Psi}{\partial t} = \Psi(x,y,z,t)E$ $\qquad\left[\because \Psi(x,y,z,t) = Ae^{-\frac{i}{\hbar}(Et-px)}\right]$

(or) $E\Psi = i\hbar\dfrac{\partial\Psi}{\partial t}$ (10)

Substituting equations (9) & (10) in equation (1) , we get

$$i\hbar\frac{\partial\Psi}{\partial t} = V\Psi - \frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}$$

(or) $i\hbar\dfrac{\partial\Psi}{\partial t} = \left[V - \dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}\right]\Psi$ (11)

Equation (11) represents the one dimensional Schrodinger time dependent wave equation along ‘x’ direction. Also the wave function Ψ (x, y, z, t) depends on both the position (x, y, z) and time (t)

Similarly for three dimensional Schrodinger time dependent wave equation can be written as

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[V - \frac{\hbar^2}{2m}\nabla^2\right]\Psi \qquad (12)$$

Where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$

Equation (12) can also rewritten as EΨ = HΨ

Where E is an energy operator given by $E = i\hbar\dfrac{\partial}{\partial t}$ &

H is called Hamiltonian operator, given by $H = V - \dfrac{\hbar^2}{2m}\nabla^2$

Schrödinger time independent wave equation

In Schrödinger time dependent wave equation the wave function ‘Ψ’ depends on time, but in Schrodinger time independent wave function ‘Ψ’ does not depend on time & hence it has many applications

We know that time dependent wave function

$$\Psi(x,y,z,t) = Ae^{-\frac{i}{\hbar}(Et-px)}$$

Now, splitting the RHS of this equation in to (i) Time dependent factor & (ii) Time independent factor, we get

$$\Psi(x,y,z,t) = Ae^{\frac{-iEt}{\hbar}} e^{\frac{ipx}{\hbar}}$$

(or) $\Psi(x,y,z,t) = A\,\psi e^{-\frac{iEt}{\hbar}}$. $\Psi(x,y,z,t) = A\psi e^{\frac{-iEt}{\hbar}}$ (1)

Where 'ψ' represents the time independent wave function. i.e., $\psi = e^{\frac{ipx}{\hbar}}$

Differentiating equation (1) partially with respect to 't' we get $\frac{\partial \Psi}{\partial t} = A\psi e^{\frac{-iEt}{\hbar}}\left[\frac{-iE}{\hbar}\right]$ (2)

Differentiating equation (1) partially with respect to 'x' we get,

$$\frac{\partial \Psi}{\partial x} = Ae^{\frac{-iEt}{\hbar}} \frac{\partial \psi}{\partial x}$$

Differentiating once again partially with respect to 'x' we get, $\frac{\partial^2 \Psi}{\partial x^2} = Ae^{\frac{-iEt}{\hbar}} \frac{\partial^2 \psi}{\partial x^2}$ (3)

We know the time dependent wave equation for 1-dimension is

$$i\hbar \frac{\partial \Psi}{\partial t} = V\Psi - \frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} \quad (4)$$

We can get the Schrödinger time dependent wave equation, just by substituting equations (1),(2) & (3), which has relation between the time dependent wave function (Ψ) and time independent wave Function (ψ) in equation (4)

Thus, substituting equations (1),(2) & (3) in equation (4) , we get

$$i\hbar A\psi e^{\frac{-iEt}{\hbar}}\left[\frac{-iE}{\hbar}\right] = VA\psi e^{\frac{-iEt}{\hbar}} - \frac{\hbar^2}{2m} Ae^{\frac{-iEt}{\hbar}} \frac{\partial^2 \psi}{\partial x^2}$$

(or) $i\hbar\psi\left[\frac{-iE}{\hbar}\right] = V\psi - \frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$

(or) $(-i)^2 E\psi = V\psi - \frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$ (or) $E\psi - V\psi = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$

(or) $\frac{\partial^2 \psi}{\partial x^2} = \frac{-2m}{\hbar^2}\left[E\psi - V\psi\right]$

(or) $\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}\left[E - V\right]\psi = 0$ (5)

Equation (5) represents the Schrodinger time independent wave function in one dimension along 'x' direction. Here the wave function is independent of time .Similarly for 3 – dimension, the Schrodinger time independent wave function is given by $\nabla^2 \psi + \frac{2m}{\hbar^2}\left[E - V\right]\psi = 0$ (6)

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Physical Significance of a wave function [Ψ]

(i) It gives the relation between the particle and wave nature of the matter statistically
i.e., $\Psi = \psi\, e^{-i\omega t}$

(ii) Wave function gives the information about the particle behavior

(iii) Ψ is a complex quantity and does not have any physical meaning

(iv) $|\psi|^2 = \dot{\psi}^* \psi$ is real & positive. This concept is similar to light. In light amplitude may be (+ve) or (-ve) but the square of intensity of light is +ve & measurable

(v) $|\psi|^2$ represents the probability density of finding the particle per unit volume

(vi) for a given volume $d\tau$, the probability of finding the particle is given by Probability (P) = $\iiint |\psi|^2\, d\tau$ where $d\tau = dx.dy.dz$

(vii) The probability will have any values between 0 & 1

1) If P =0 , then there is no particle within the given limits

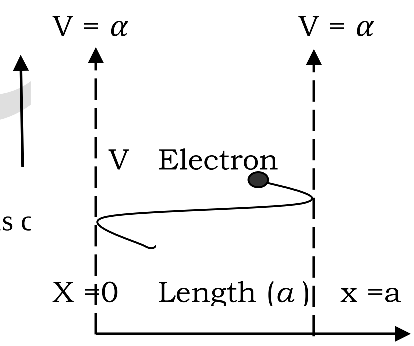
2) If P = 1 ,the particle is definitely present within the given limits

3) If P = 0.7, then there is 70% chance of finding the particle within the given limits. Also there is 30% of no chance of finding the particle

4. Using Schrodinger's time independent wave equation normalize the wave function of electron trapped in a one dimensional potential well?

Let us consider a particle (electron) of mass 'm' moving along x- axis, enclosed in a one dimensional potential box as shown in figure 4.14. Since the walls are of infinite potential the

Particle does not penetrate out from the box



Also, the particle is confined between the lengths 'a' of the box and has elastic collisions with the Walls. Therefore the potential energy of the electron inside the box is c taken as zero for simplicity

∴ Outside the box and on walls of the box, the potential energy V of the electron is α. Inside the box the potential energy of the electron is zero

i.e., the boundary condition is $V(x) = 0\ when\ 0 < x < a$

$$V(x) = \alpha\ when\ 0 \geq x \geq a$$

Since the particle cannot exist outside the box and thus wave function ψ = 0 at $0 \geq x \geq a$

Now, Consider the Schrodinger one dimensional time independent wave function

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}[E - V]\psi = 0$$

Since the potential energy inside the wall is zero, the particle has kinetic energy alone. Hence it is called free particle (electron), now the above equation becomes

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}E\psi = 0$$

(or) $\frac{\partial^2\psi}{\partial x^2}+k^2\psi=0$ (1)

Where $k^2=\frac{2mE}{\hbar^2}$ (2)

The second order differential equation of equation (1) has two arbitrary constants

∴ The solution of equation (1) is $\psi(x)=A\sin kx+B\cos kx$ (3)

Where A & B are arbitrary constants which can be found by applying the boundary conditions

Condition (i) at x =0, potential energy V = α. Hence there is no particle at the walls of the box, therefore ψ(x) = 0 Equation (3) becomes $0=A\sin 0+B\cos 0$

$= \quad 0 \quad + B(1)$

$\therefore B = 0$

Condition (ii) at x = a, potential energy V =α there is no particles at the walls of box ∴ ψ(x) = 0

Now, Equation (3) becomes $0=A\sin ka+B\cos ka$

(or) $0=A\sin ka+0$ [∵B =0 from condition (i)]

Also A is a Constant & hence $A\neq 0; \sin ka=0$

Thus, we can write as $\sin n\pi=0$

Comparing these two equations we can write $ka=n\pi$, where '*n*' is a integer

(or) $k=\frac{n\pi}{a}$ (4)

Substituting the value of B & k in equation (3)

The wave function in one dimensional box is $\psi(x)=A\sin\left(\frac{n\pi x}{a}\right)$ (5)

Energy of the particle (electron)

Equation (2) can be rewritten as $k^2=\frac{2mE}{\left(\frac{h^2}{4\pi^2}\right)}$ $\left[\because \hbar=\frac{h}{2\pi}\right]$

(or) $k^2=\frac{8\pi^2 mE}{h^2}$ (6)

Squaring equation (4), we get $k^2=\frac{n^2\pi^2}{a^2}$ (7)

Equating equation (6) & (7) we get

$$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{a^2}$$

$\therefore$ The Energy of the particle (electron) $E_n = \frac{n^2h^2}{8ma^2}$ (8)

From equation (5) & (8) we can say that for each value of 'n' there is energy
Level with the corresponding wave function and hence each E_n is said to be *Eigen value* corresponding to the *Eigen function* ψ_n

Energy levels of an electron:

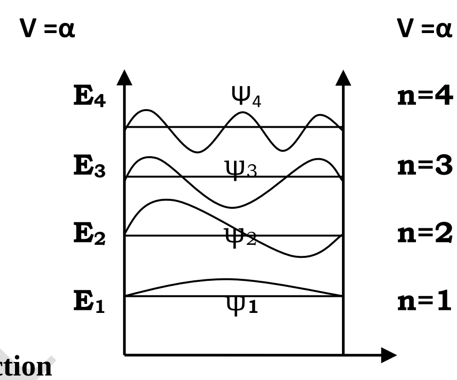
The ground energy state can be calculated by substituting n = 1 in equation (8), we get, $E_1 = \frac{h^2}{8ma^2}$

For n=2, $E_2 = \frac{2^2h^2}{8ma^2} = 4E_1$; For n=3, $E_3 = \frac{3^2h^2}{8ma^2} = 9E_1$, etc.,

Similarly we can calculate 'n' number of energy levels by substituting n=1, 2, 3… n.

In general we can write $E_n = n^2E_1$ $E_n = n^2E_1$ (9)

From these levels, it is found that each energy level of an electron is discrete
The various Eigen values corresponding to their Eigen function is shown in figure 4.15(a)



Normalization of the wave function

It is process by which the probability (P) of finding the particle (electron) inside the box
We know that if (P) = 1 then the particle lies inside the box

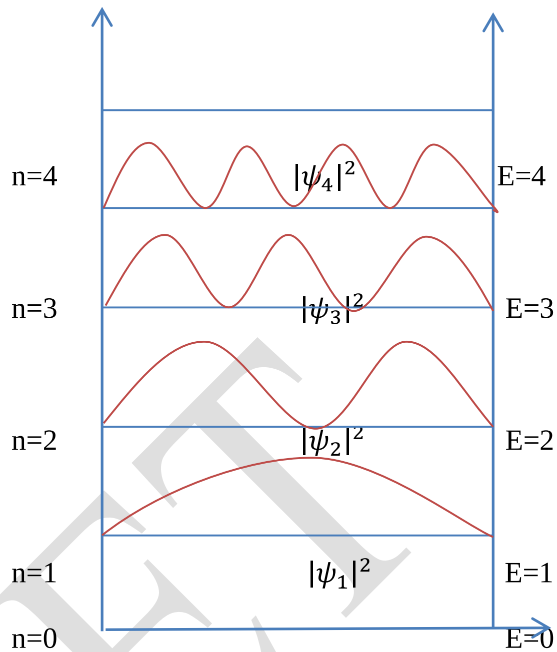
$\therefore$ Probability of 1 – D box of length 'l' is $P = \int_0^a |\psi^2| dx = 1$ (10)

[$\because$ *the particle lies between* 0 *and* l]

Substituting equation (5) in equation (10) we get

$$P = \int_0^a A^2 \sin^2 \frac{n\pi x}{a} dx = 1$$

$$(\text{or } P = A^2 \int_0^a \left\{ \frac{1 - 2\cos\left(\frac{n\pi x}{a}\right)}{2} \right\} dx = 1$$



$$(\text{or})\ \ P = A^2 \left[\left(\frac{x}{2}\right) - \left(\frac{1}{2}\right) \frac{\sin\left(\frac{2\pi nx}{a}\right)}{\left(\frac{2n\pi}{a}\right)} \right]_0^a = 1$$

$$(\text{or})\ \ P = A^2 \left[\left(\frac{a}{2}\right) - \left(\frac{1}{2}\right) \frac{\sin\left(\frac{2\pi na}{a}\right)}{\left(\frac{2n\pi}{a}\right)} \right] = 1$$

$$(\text{or})\ \ \ P = A^2 \left[\left(\frac{a}{2}\right) - \left(\frac{1}{2}\right) \frac{\sin(2\pi n)}{\left(\frac{2n\pi}{a}\right)} \right] = 1 \qquad (11)$$

We know that sin nπ = 0 ∴ $\sin 2n\pi = 0$

Equation (11) can be rewritten as $\frac{A^2 a}{2} = 1$

(or) $A^2 = \frac{2}{a}$

(or) $A = \sqrt{\frac{2}{a}}$

Substituting the value of ‘A’ in equation (5), we get

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \qquad (12)$$

Equation (12) is said to be *normalized wave function*

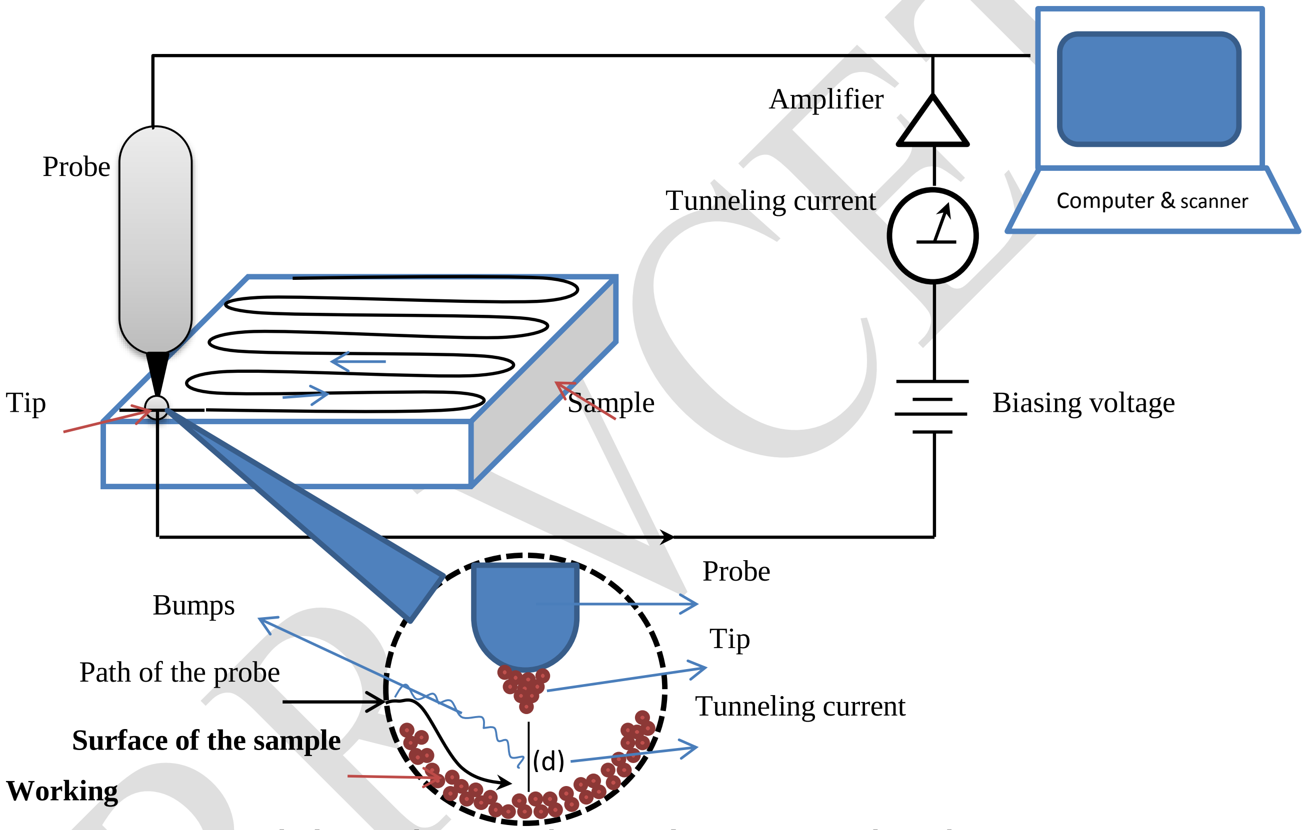
5. Write a brief note about the working mechanism of Scanning Tunneling Electron Microscope with necessary diagrams?

Principle

The tunneling of electron between the sharp metallic tip of the probe and the surface of the sample. Here the tunneling current is maintained by adjusting the distance between the tip and the sample, with an air gap for electron to tunnel. IN a similar way the tip is used to scan atom by atom and line by line of the sample and the topography of the sample is recorded in the computer

Construction

(i) The experimental setup consists of a probe in which a small thin metal wire is etched in such a way that the tip of the probe will have only one atom as shown in figure.
(ii) The tip is tapered down to a single atom, so that it can follow even a small change in the contours of the sample.
(iii) The tip is connected to the scanner and it can be positioned to X, Y, Z coordinates using a personal computer, as shown in figure.
(iv) The sample for which the image has to be recorded is kept below the tip of the probe at a particular distance (atleast to a width of 2 atoms spacing) in such a way that the tip should not touch the sample. i.e., A small air gap should always be maintained between the tip of the probe and the sample.
(v) The computer is also used to record the path of the probe and the topography of the sample in a grey-scale (or) colour. Necessary circuit connections along with an amplifier are provided to measure the tunneling current in the circuit



Working

1. Circuit is switched ON and necessary biasing voltage is given to the probe.
2. Due to biasing the electrons will tunnel (or) jump between the tip of the probe and the sample and therefore produces a small electric current called tunneling current as show in figure.
3. The tunneling current flows through the circuit only if the tip is in contact with the sample through the small air gap at a distance '*d*' between them.
4. The current produced is amplified and measured in the computer
5. It is found that the current increases (or) decreases based on the distance between the tip of the probe and the sample
6. The current in the circuit should be monitored in such a way that it should be maintained constant
7. Therefore, for maintaining the constant current, the distance (*d*) between the tip and the sample should be continuously adjusted, whenever the tip moves over the surface of the sample.

8. The height fluctuations (*d*) between the tip and the sample is accurately recorded and as a resultant, a map of '*bumps*' is obtained in the computer as shown in figure.
9. In a similar way the tip is scanned atom by atom and line by line of the sample and the topography of the sample is recorded in the computer.
10. The STM does not show the picture of the atom, rather it records only the exact only the exact position of the atoms, more precisely the position of electrons.

Advantages

1. It can scan the positions & topography atom by atom (or) even electrons
2. It is the latest technique used in research laborites for scanning the materials
3. very accurate magnification up to nan-scale shall be measured.

Disadvantages

1. Even a very small vibrations will deviate the measurement setup
2. it should kept in vacuum, because even a single dust particle will damage the tip of the probe
3. cost is high

Applications

1. it is used to produce integrated circuit
2. it is used in biomedical devices
3. they are used in materials science studies for both bump and flat surfaces

6. What are matter waves? Give its properties. Write the brief note on experimental verification of matter's waves using G.P. Thomson experiment?

According to de-Broglie hypothesis, a moving particle is always associated with waves.

(i) Waves and particles are the only two modes through which energy can propagate in nature
(ii) Our universe is fully composed of light radiation and matter
(iii) Since nature loves symmetry, so matter and waves must be symmetric.

The waves associated with the matter particles are called *matter waves or de-Broglie waves.*

From Planck's theory, the energy of a photon of frequency υ is given by $E = h\,\upsilon$ (1)

According to Einstein's mass energy relation, $E = mc^2$ (2)

Where m – mass of a photon, *c* – velocity of a photon

Equating (1) and (2), we get

$$h\,\upsilon = mc^2 \quad (3)$$

$$\frac{hc}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mc} \quad (4)$$

Since $mc=p$ momentum of photon, then

$$\lambda = \frac{h}{p} \quad (5)$$

According to de-Broglie hypothesis, the wavelength of de-Broglie wave associated with any moving particle of mass *'m'* with velocity *'v'* is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad (6)$$

In terms of Energy

We know that K.E (E)= $\frac{1}{2}mv^2$

Multiply *m* by both sides mE= $\frac{1}{2}m^2v^2$

(or) $\sqrt{2mE} = mv$

(or) $\sqrt{2mE} = p$

We know that, $\lambda = \frac{h}{p}$ and hence

(or) de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mE}}$

In terms of electrons

We know that kinetic energy in terms of electron volt is given by eV= $\frac{1}{2}mv^2$

Multiply *m* by both sides eV= $\frac{1}{2}m^2v^2$

(or) $\sqrt{2meV} = mv$

(or) $\sqrt{2meV} = p$

We know that, $\lambda = \frac{h}{p}$ and hence

(or) de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2meV}}$

Properties of Matter

(ix) Matter waves are not electromagnetic waves.

(x) Matter waves are new kind of waves in which due to the motion of the charged particles, electromagnetic waves are produced.

(xi) Lighter particles will have high wavelength

(xii) Particles moving with less velocity will have high wavelength

(xiii) The velocity of matter wave is not a constant, it depends on the velocity of the particle.

(xiv) If the velocity of the particle is infinite then the wavelength of matter wave is indeterminate(λ=0)

(xv) The wave and particle aspects cannot appear together

(xvi) Locating the exact position of the particle in the wave is uncertain

Experimental verification of matter waves – G.P. Thompson's Experiment

Construction & Working

A high energy electron beam is produced by the cathode C. The beam is excited with potentials up to 50,000 volts. A fine pencil beam is obtained by passing it through the slit or diaphragm S. The accelerated fine beams of electrons are made to fall on a thin film of gold G or aluminum of the order of 10^{-6} cm. The photograph of the beam from the foil is recorded on a photographic plate P.

The whole apparatus is exhausted to a high vacuum so that the electrons may not lose their lose their energy in collision with the molecules of the gas. After developing the photographic plate, the resultant diffracted pattern obtained as shown in figure

Since ordinary metals like gold are microcrystalline in structure, the diffracted electrons produced by them are similar in appearance to the X – ray diffraction powder patterns and consists of a series of well-defined concentric rings about a central spot as shown in figure. To make sure that this pattern is due to the electrons and not due to any possible X – rays generated, the cathode rays in the discharge tube are deflected by a magnetic field. It was observe that the whole diffracted pattern, observed on the fluorescent screen placed instead of photographic plate also shifted. Once it is confirmed that the diffraction pattern is due to the electrons. Thomson calculated the wavelength of the deBroglie waves associated with the cathode rays and determined the spacing between the atomic planes in the foils using Bragg's equation. He obtained results in good agreement with those from X – ray studies.