

5. Nano Devices

5.1. Introduction

A nanometer is one billionth ($1/10^9$) of a meter. For comparison, thickness of a single human hair is about 80,000 nm (80 μm), a red blood cell is approximately 7000 nm (7 μm) wide and a water molecule is almost 0.3nm across. Scientist and engineers are now-a-days interested in Nanoscale which is from 1 nm to 100 nm. At Nanoscale, the properties of materials are very different from those at larger scale. Therefore, the nano-world is in between quantum world and macro world.

Nanoscience

It is concerned with the study of phenomena and application of structures, devices and systems by controlling shape and size at the nanometer scale. Nanotechnology means making use of the unique physical properties of atoms, molecules and other materials measuring roughly 1 to 100 nanometre. The word “nano” comes from nanos, a Greek word meaning dwarf. Presently, we are making devices made of nanoelectronic devices. The microelectronics industry was born out of the invention of the bipolar transistor in 1947 and by the invention of the integrated circuit (IC) in 1958. Gordon Moore (INTEL founder) observed that the number of transistor per square inch on IC chip roughly doubled by every 18 to 24 months. This general rule of thumb is now called as “**Moore’s law**”. In 1960, the minimum size of a transistor was approximately 100 nm. At present, manufacturing technology is at transistor size of 22 nm. Because of the diminishing feature size of transistors and other components, we can say that the electronics industry is already doing “nanotechnology”.

Nanomaterials

Definition

Nanophase materials are newly developed materials with grain size at the nanometer range (10^{-9}m). ie., in the order of 1 -100 nm. The particle size in a nanomaterial is 1 – 100 nm. They are simply called nanomaterials.

Different forms of Nanomaterials

Nanostructured material

The structures whose characteristic variations in design length is at the Nanoscale

Nanoparticles

The particle size is in the order of 10^{-9}m are called nanoparticles

Nano dots

Nanoparticles which consist of homogeneous material, especially those that are almost spherical or cubical in shape

Nano rods

Nanostructures which are shaped like long sticks or rods with diameter in Nanoscale and a length very much longer.

Nanotubes

Nanotubes are Nanoscale materials that has a tube like (hollow cylinder) structure.

Nanowires

Nanowires are solid rod like material with diameter of few nanometers or less

Fullerenes

A form of carbon having a large molecule consisting of an empty cage of 60 or more carbon atoms.

Nanocomposites

Composite structures whose characteristics dimensions are foun at Nanoscale

Cluster

A collection of units (atoms or reactive molecules) upto few tens of units.

Colloids

A stable liquid phase containing particles in the 1 – 1000 nm range.

Nanoelectronics

It refers to the use of nanotechnology in electronic components, especially transistors. It often refers to transistor devices that are so small that interatomic interactions and quantum mechanical properties need to be studied extensively. Besides, being small and allowing more transistors to be packed into a single chip, the uniform and symmetrical structure of nanotubes allows a higher electron mobility, a symmetrical electron/hole characteristic.

Need for Nanotechnology in electronics

Today microelectronics are used to solve most of the problems. The two exceptional disadvantages of microelectronics are: Physical size, increasing cost of fabrication of ICs. To overcome these disadvantages, nanotechnology is used.

Advantages of using Nanotechnology in Electronics

Increasing the density of memory chips.

Decreasing the weight and thickness of the screens

Nanolithography is used for fabrication of chips.

Reducing the size of transistors used in integrated circuits.

Improving display screen on electronic devices.

Reducing power consumption.

5.2. Electron density in bulk material

The bulk material is a collection of atoms having properties that are from individual atoms. The nanomaterials gives unique electronic properties. One of the major difference in nanomaterials with respect to bulk materials is the number of available energy states. In a bulk material, the states within each energy sublevel are so close that they blend into a band.

The total number of electron states N with energies up to E , can be determined based on the

$$N = \frac{\pi}{3} \left[\frac{8m}{h^2} \right]^{3/2} E^{3/2} \cdot V \quad (1)$$

Here, we represent the volume as V , m is the mass of an electron and h is the Planck's constant.

$$\text{The number of energy states per unit volume is given by } n = \frac{N}{V} = \frac{\pi}{3} \times \left[\frac{8m}{h^2} \right]^{3/2} E^{3/2} \quad (2)$$

Density of states is defined as number of available electron energy states per unit volume, per unit energy i.e., $Z(E) = dn / dE$ (3)

Hence equation (2) becomes,

$$Z(E) = \frac{\pi}{3} \times \left(\frac{8m}{h^2} \right)^{3/2} \times \frac{3}{2} E^{1/2}$$

$$(\text{or}) \quad Z(E) = \frac{\pi}{2} \times \left(\frac{8m}{h^2} \right)^{3/2} E^{1/2} \quad (4)$$

From equation (4), the density of states for a bulk material is directly proportional to square root of energy

$$\text{i.e., } Z(E) \propto \sqrt{E} \quad (5)$$

The relevant application of density of states is that it provides information about nanomaterials.

Here, the fermi function gives the probability of occupation by the free electrons in a given energy state.

$$\text{i.e., } f(E) = \frac{1}{1 + e^{\frac{E - E_f}{kT}}} \quad (6)$$

$$\text{Then, the number of free electrons per unit volume is } n_e = \int_0^{\alpha} F(E) Z(E) dE$$

$$\text{Put } F(E) = 1 \text{ at } T = 0\text{K, then } n_e = \frac{\pi}{2} \times \left(\frac{8m}{h^2} \right)^{3/2} \int_0^{\alpha} E^{1/2} dE$$

$$(\text{or}) \quad n_e = \frac{\pi}{3} \times \left(\frac{8m}{h^2} \right)^{3/2} E_f(0)^{3/2} \quad (7)$$

5.3. Size dependence of Fermi energy

In terms of the distribution of energy, solid have thick energy bands, whereas atoms have thin, discrete energy states. Hence to make a solid behave electronically more like an atom, we need to make it about the same size as an atom.

Hence rearranging equation (7) , we get $E_f(0) = \frac{h^2}{8m} \times \left(\frac{3n_e}{\pi} \right)^{2/3}$ (8)

In the above equation, 'n' is the only variable.

Equation (8) suggests that the fermi energy of a conductor depends on the number of free

electrons 'N' per unit volume 'V' $E_F(0) \propto (n)^{2/3} \propto \left(\frac{N}{V} \right)^{2/3}$ (9)

Since the electron density is a property of the material, the fermi energy does not vary with material's size. E_F is same for a particle or for a brick of copper. Hence the energy state will have the same range for small volume and large volume of atoms. But for small volume of atoms we get larger spacing between states. This is applicable to semiconductors and insulators.

Let us consider that all states up to $E_F(0)$ are occupied by a total of free electrons (N).

$$\Delta E = \frac{E_F(0)}{N} \quad (10)$$

$$\text{From equation (9) \& (10), } \Delta E \propto \frac{1}{V} \quad (\text{or}) \quad \Delta E \propto \frac{1}{a^3} \quad (11)$$

Thus, the spacing between energy states is inversely proportional to the volume of the solid. The energy sublevel and the spacing between energy states within it will depend on the number of atoms as shown in figure. At one point, we know that an energy sublevel must be divided as many times as there are atoms in a solid, which eventually results too many splits to differentiate. Hence, we just refer to each sublevel as a solid energy band. On the other hand, a single atom in the sublevel contain only one discrete energy state. If we reduce the volume of a solid, the tiny piece of material behaves electronically like an artificial atom.

5.4. Quantum confinement

Definition

It is a process of reduction of the size of the solid such that the energy levels inside becomes discrete

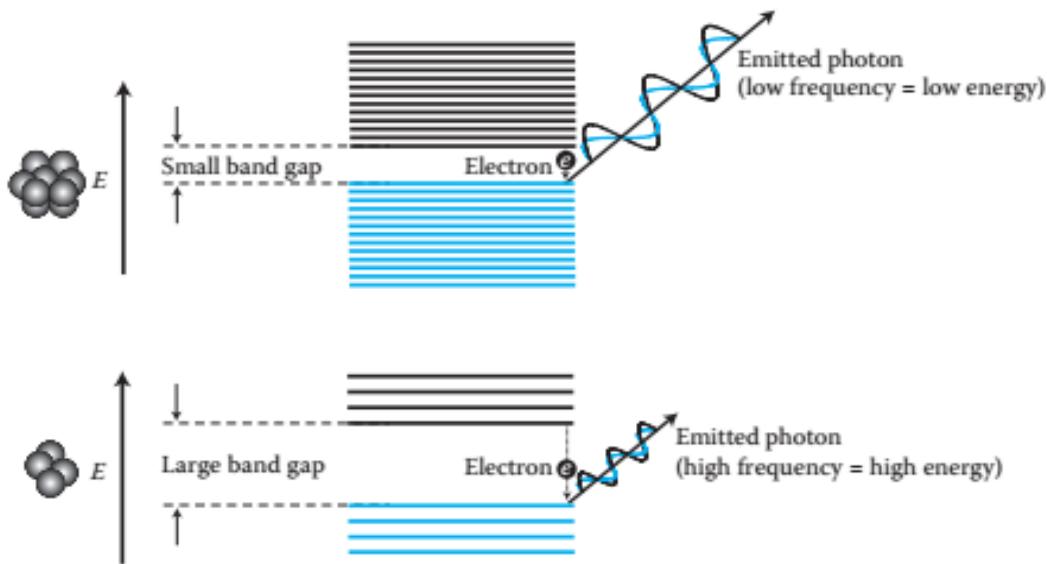
Explanation

When the size of a nanocrystal becomes smaller than the deBroglie wavelength, electrons and holes gets spatially confined, electrical dipoles gets generated, the discrete energy levels are formed. As the size of the material decreases, the energy separation between adjacent levels increases. The density of states of nanocrystals is positioned in between discrete (as that of atoms and molecules) and continuous (as in crystals).

Quantum size effect is most significant for semiconductor nanoparticles. In semiconductors, the bandgap energy is of the order of few electron volts and increases with a decrease in particle size. When photons of light fall in a semiconductor, only those photons with energy are absorbed and a sudden rise in absorption is observed when the photon energy is equal to the bandgap.

As the size of the particle decreases, absorption shifts towards the shorter wavelength (blue shifts) indicating an increase in the bandgap energy .A change in absorption causes a change in the colour of the semiconductor nanoparticle.

For example, bulk cadmium sulphide is orange in colour and has a bandgap of 2.42eV . It becomes yellow and then ultimately white as its particle size decreases and the bandgap increases.



5.5. Quantum structures

Definition

When a bulk material is reduced in its size, atleast one of its dimension, in the order of few nanometers, then the structure is known as quantum structure.

Explanation

The volume of a box can be reduced by shortening its length, width or, and height. The same is true for the region occupied by the electrons in a solid. There are three dimensions to confine the bulk material. The quantum confinement needs confining at least one of these dimensions to less than 100 nanometers or even just a few nanometers.

The more the dimensions are confined, the more the density of states function looks like that of an atom. This progressive discretization gives new ways to understand real atoms, behaviour of electrons and developing quantum confined electronic devices.

A structure in which the motion of the electrons or holes are confined in one or more directions by potential barriers is called quantum confined structure.

The quantum confined structure is classified into three types based on the confinement directions. They are

- (i) Quantum well
- (ii) Quantum wire
- (iii) Quantum dot

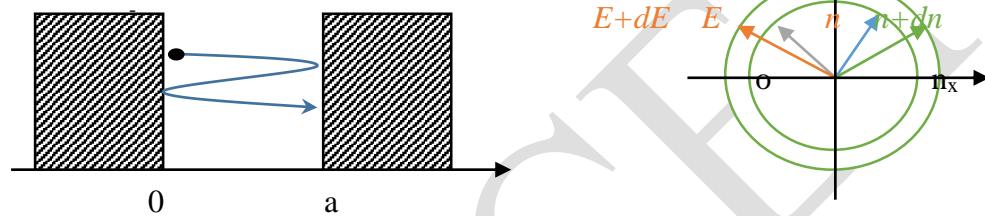
The density of states of a bulk material is given by

$$Z(E) = \frac{8\pi\sqrt{2}(m^*)^{3/2}(E - E_c)^{1/2}}{h^3}$$

Where E_c is the bottom of conduction band energy & m^* is the effect mass of the electron.

Quantum well (2D)

The quantum well can be displayed with dimensions of length a , where the electrons of effective mass are confined in the well as shown in fig.



The two dimensional density of states is the number of states per unit area and unit energy. Consider the electron in a two dimensional bounded region of space. We want to find how many quantum states lie within a particular energy, say, between E and $E+dE$ as shown in Figure.

The reduced phase space now consists only the x - y plane and n_x and n_y coordinates.

In 2D space, $n^2 = n_x^2 + n_y^2$

Derivation

The number of available states within a circle of radius ' n ' is given by $\frac{1}{4}\pi n^2$

Here only one quarter of circle will have positive integer values

The number of states within a circle of radius $n+dn$ is given by $\frac{1}{4}\pi(n+dn)^2$

The number of available energy states lying in an energy interval E and $E+dE$

$$\begin{aligned} Z(E)dE &= \frac{1}{4}\pi[(n+dn)^2 - n^2] \\ &= \frac{\pi}{4}[n^2 + dn^2 + 2ndn - n^2] \end{aligned}$$

As dn^2 is very small, we can neglect dn^2 . Therefore we get,

$$Z(E)dE = \frac{\pi}{4}[2ndn] = \frac{\pi}{2}ndn \quad (1)$$

$$\text{We know that } n^2 = \frac{8m^*E}{h^2}a^2 \quad (2)$$

$$(or) n = \left[\frac{8m^* E}{h^2} \right]^{1/2} a \quad (3)$$

$$(or) dn = \left[\frac{8m^*}{h^2} \right]^{1/2} a \frac{1}{2} E^{-1/2} dE \quad (4)$$

Substitute the value of equation (3) and (4) in equation (1), we get

$$Z'(E)dE = \frac{\pi}{2} \left[\frac{8m^* E}{h^2} \right]^{1/2} a \left[\frac{8m^*}{h^2} \right]^{1/2} a \frac{1}{2} E^{-1/2} dE$$

m^* is the effective mass in the quantum well

$$Z'(E)dE = \frac{\pi}{4} \left[\frac{8m^*}{h^2} \right] a^2 dE \quad (5)$$

Put $a^2 = A$ area of circle

According to Pauli's exclusion principle each energy level can occupy two electrons of opposite spin

$$i.e., Z'(E)dE = 2 \times \frac{\pi}{4} \left[\frac{8m^*}{h^2} \right] AdE$$

Number of quantum states per unit area and unit energy is

$$\frac{Z'(E)dE}{AdE} = 2 \times \frac{\pi}{4} \left[\frac{8m^*}{h^2} \right]$$

$$Z'(E) = \frac{\pi}{2} \left[\frac{8m^*}{h^2} \right] \quad (or) \quad Z'(E) = \frac{\pi}{2} \left[\frac{8m^*}{(2\pi\hbar)^2} \right] \quad [\text{since } h^2 = 4\pi\hbar^2] \quad (6)$$

The density of states in two dimensional is given by $Z'(E)^{2D} = \frac{m^*}{\pi\hbar^2}$ for $E \geq E_0$ (7)

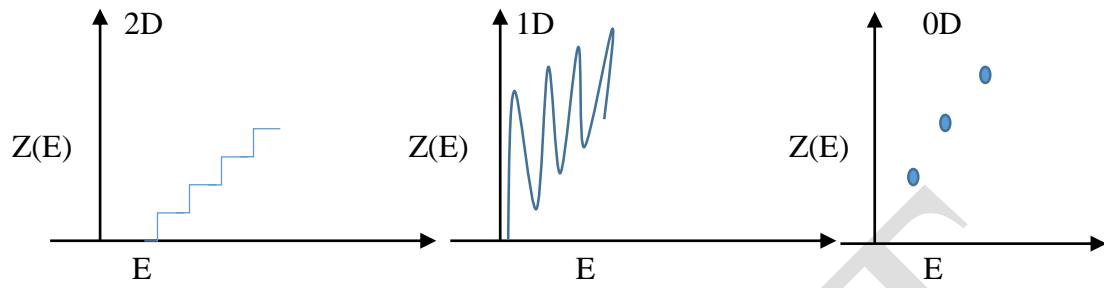
Where E_0 is the ground state of quantum well

$$Z'(E)^{2D} = \frac{m^*}{\pi\hbar^2} \sum_n \sigma(E - E_n) \quad (8)$$

Where E_n are the energies of quantized states and $\sigma(E - E_n)$ is step function.

From equation (7), the density of states in two dimension is constant with respect to the energy.

i.e., $Z'(E)^{2D} \propto E^0 = \text{constant}$



Density of states in quantum wire (1D)

Consider the one dimensional system, the quantum wire in which only one direction of motion is allowed. (eg. Along x – direction).

In one dimension, such as for a quantum wire, the density of states is defined as the number of available states per unit length per unit energy around an energy E . The electron inside the wire are confined in a one dimensional infinite potential well with zero potential inside the wire and infinite potential outside the wire.

At $x = 0$; $V(x) = 0$ for an electron inside the wire

At $x = a$; $V(x) = \alpha$ for an electron outside the wire

The reduced phase space now consists only the x plane and n_x coordinates are shown in figure.

In one dimensional space $n^2 = n_x^2$

The number of available energy states lying in an interval of length is

$$Z'(E)dE = n + dn - n = dn \quad (1)$$

Substitute the value of dn from equation (4), we get

$$Z'(E)dE = \left[\frac{8m^*}{h^2} \right]^{1/2} a \frac{1}{2} E^{-1/2} dE \quad (2)$$

According to Pauli's exclusion principle, two electrons of opposite spin can occupy each energy state.

$$Z'(E)dE = 2 \times \left[\frac{8m^*}{h^2} \right]^{1/2} a \frac{1}{2} E^{-1/2} dE$$

Number of quantum states per unit length and unit energy is $\frac{Z'(E)dE}{adE} = \left[\frac{8m^*}{h^2} \right]^{1/2} E^{-1/2}$

$$(\text{or}) \quad Z'(E) = \left[\frac{8m^*}{4\pi^2 \hbar^2} \right]^{1/2} E^{-1/2} = Z'(E)^{1D} = \left[\frac{2m^{*1/2}}{\pi \hbar} \right] E^{-1/2} \quad (3)$$

$$\text{If the electron has potential energy } E_0 \text{ we have } Z(E)^{1D} = \frac{1}{\pi \hbar} \sqrt{\frac{2m^*}{(E - E_0)}} \quad (E \geq E_0) \quad (4)$$

From equation (4) the density of states in one dimensional system has a functional dependence on energy $Z(E)^{1D} \propto E^{-1/2}$

For more than one quantized state, the one dimensional density of states is given by

$$Z(E)^{1D} = \frac{1}{\pi \hbar} \sum \sqrt{\frac{2m^*}{(E - E_0)}} \sigma(E - E_n) \quad (5)$$

Where E_n are the energies of the quantized states of the wire and $\sigma(E - E_n)$ is the step function. The density of states in quasi-continuum (or) quantum wire is shown in figure. The discontinuities in the density of states are known as **Van Hove Singularities**

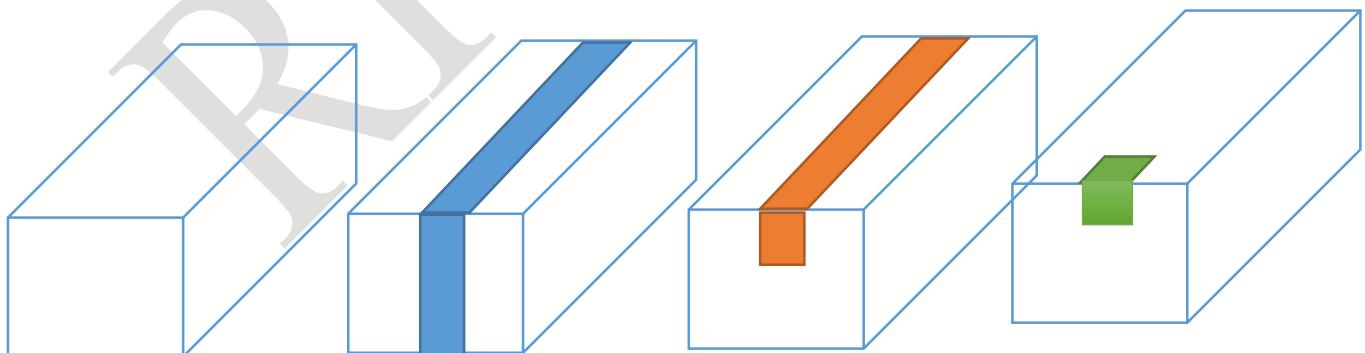
Density of states in Quantum dot (0D)

In a zero dimensional system, the density of states are truly discrete and they don't form a quasi continuum.

In zero dimensional system (quantum dot), the electron is confined in all three spatial dimensions and hence motion of electron is possible. Each quantum state of a zero dimensional system can therefore be occupied by only two electrons. So the density of states for a quantum dot is merely a delta function.

$$Z(E)^{0D} = 2\sqrt{(E - E_0)} \quad (6)$$

Here, the factor 2 accounts for spin. For more than one quantum state, the density of states is given by $Z(E)^{0D} = \sum_n 2\delta\sqrt{(E - E_0)}$



Bulk

Quantum well

Quantum wire

Quantum dot

Confinements

No

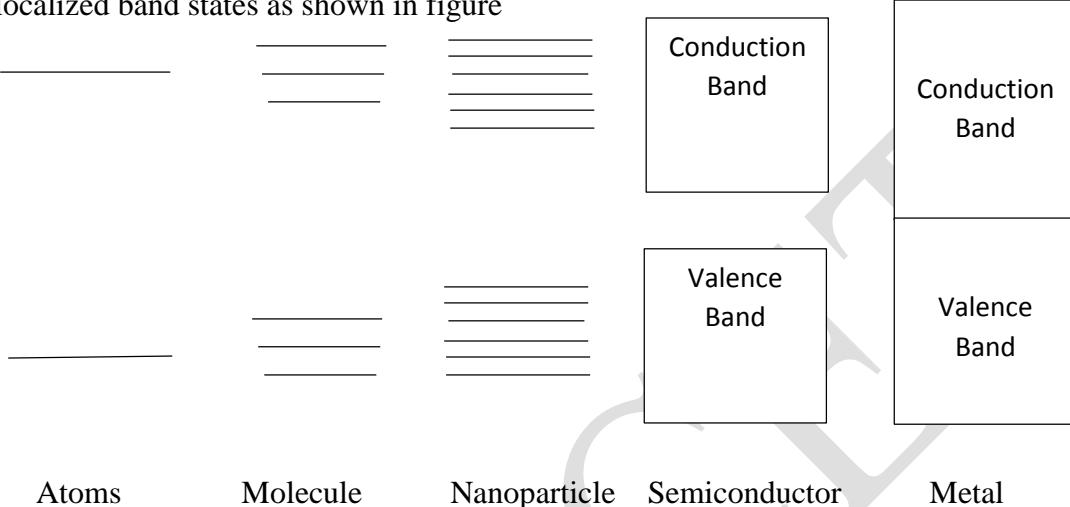
1D

2D

3D

5.6. Bandgap of Nanomaterials

The electronic properties of metals and semiconductors are determined by their electronic band structure. The band structure changes with particle size. Molecular orbitals get converted into delocalized band states as shown in figure



The band structure of nanocrystals lies between the discrete density of states as in atoms and molecules continuous band as in crystals. As the size of the material decreases, the energy separation between the adjacent levels increases. This size quantization effect is responsible for the transition of electronic states from a bulk material or semiconductor to nanoparticles. The particles that show this size quantization effect are called Q-particles or quantum dots.

In case of the particle size being less than the deBroglie wavelength, charge carriers can be quantum-mechanically understood as particles in a box and the size of the box can provide the dimensions of the particle. With a decrease in particle size of metals, the quasi-continuous density of states splits into discrete electronic levels with an increase in spacing between these levels.

Quantum size effect is most significant for semiconductor nanoparticles. In semiconductor, a bandgap already exists in the bulk state. It also increases and the energy bands gradually convert into discrete molecular electronic levels with a decrease in particle size. As the size of metal nanoparticles decreases, they tend to lose their metallic character and become semiconductor. In metals, the quantum size effect exists but it can be seen only in particles smaller than 2nm where localization of energy levels can be observed when the spacing between the levels exceeds thermal energy (about 26 MeV).

5.7. Tunneling

The phenomenon of penetration of charge carriers directly through the potential barrier, instead of climbing on it is called tunnelling.

Single electron phenomena

Transistors are what computers used to compute-tiny switches turning on and off, transferring and amplifying signals, making logic decisions. Today, microchips have over a billion transistors, each one turning on and off a billion times every second. These chips require manufacturing processes with roughly 100-nm resolution. And every year this resolution drops, enabling even smaller transistors, so that even more of them can be squeezed into the same amount of space. Rather than moving torrents of electrons through transistors, it may very well be practical and necessary to move electrons *one at a time*. We can use transistors to make sensitive amplifiers, electrometers, switches, oscillators, and other digital electronic circuits all of which operate using single electrons

Rules for single electron phenomena to occur

Tunnelling is the way electrons cross both the physical barriers and the energy barriers separating a quantum dot from the bulk material that surrounds it. If any electron on one side of the barrier could just tunnel across it, there would not be any isolation. The dot would not be a quantum dot because it would still essentially be part of the bulk.

So we need to be able to control the addition and subtraction of electrons. We can do this with voltage biases that force the electrons around. There are two rules for preventing electrons from tunnelling back and forth from a quantum dot.

- (i) Coulomb blockade effect
- (ii) Overcoming uncertainty

Rule1: Coulomb Blockade effect

A quantum dot has a capacitance, C_{dot} , a measure of how much electric charge it can store
 $C_{dot} = G \epsilon d$ (1)

Here, ϵ is the permittivity of the material surrounding the dot, d is the diameter of the dot, and G is a geometrical term (if the quantum dot is a disk, $G = 4$; if it is a spherical particle, $G = 2\pi$). An object isolated in space can store charge on its own and therefore can have a capacitance.

The energy needed to add one negatively charged electron to the dot is known as the charging

$$\text{energy, } E_C = \frac{e^2}{2C_{dot}} \quad (2)$$

We know that the coulomb blockade can prevent unwanted tunnelling. Hence we can keep the quantum dot isolated, the condition for this is given by $E_C \gg K_B T$ (3)

Rule2: Overcoming uncertainty

The uncertainty in the energy of a system is inversely proportional to how much time we have to measure it. Specifically, the energy uncertainty, ΔE , adheres to this relationship

$$\Delta E \approx \frac{\hbar}{\Delta t} \quad (4)$$

Here, h is Planck's constant and Δt is the measurement time. Since it is a tiny capacitor, the time we use for Δt is the capacitor's time constant (the characteristic time a capacitor takes to acquire most of its charge). The time constant of a capacitor is RC , where R is the resistance and C is the capacitance. In our case, the resistance is the tunnelling resistance, R_t , and the capacitance is C_{dot} . This gives us $\Delta t = R_t C_{dot}$ (5)

Our goal is to keep electrons from tunnelling freely back and forth to and from the dot. To ensure this, *the uncertainty of the charging energy must be less than the charging energy itself*.

For maintaining electron isolation in quantum dot, we need $\Delta E_c < E_c$ (6)

Substituting equation (2), (4) and (5) in (6), we get $\frac{h}{R_t C_{dot}} < \frac{e^2}{2C_{dot}}$ (7)

In otherwords, $R_t \gg \frac{h}{e^2}$ (8)

Meeting this criterion is often as simple as making sure the insulating material surrounding the dot is thick enough. These two rules help in building a single-electron transistor (SET)

5.8. Single electron transistor (SET)

Principle

A transistor with three terminal switching device made from a quantum dot that controls the current from source to drain one electron at a time is called single electron transistor

Construction

The single electron transistor (SET) is built like a conventional Field Emitting Transistor (FET). It has tunnelling junctions in place of pn – junctions and quantum dot in place of the channel region of the FET. To control tunnelling, a voltage bias to the gate electrode is applied. A separate voltage bias is applied between source and drain electrodes for the current direction. For current to flow, gate bias voltage must be large enough to overcome the coulomb blockade energy.

Working

1. The purpose of SET is to individually control the tunnelling of electrons into and out of the quantum dot. To do this, we must first stop random tunnelling by choosing the right circuit geometry and materials. If an electron comes or goes from the dot. It will on purpose
2. To control tunnelling, we apply a voltage bias to the gate electrode. There is also a voltage difference between the source and the drain that indicates the direction of current. Here, we can say that current and electron flow in the same direction and we will consider the electrode from which the electrons originate.
3. This is similar to the working of FET, where the gate voltage creates an electric field that alters the conductivity of the semiconducting channel below it, enabling current to flow from source to drain.
4. Applying a voltage to the gate in an SET creates an electric field and change the potential energy of the dot with respect to the source and drain. This gate voltage controlled potential

difference can make electrons in the source attracted to the dot and simultaneously electrons in the dot attracted to the drain.

5. For current to flow, this potential difference must be atleast large enough to overcome the energy of the coulomb blockade.

The energy “ E ” needed to move a charge e across a potential difference V is given by $E=Ve$

So, the voltage that will move an electron onto or off the quantum dot is given by

$$V = \frac{E_c}{e} \quad (\text{or}) \quad V = \frac{e^2}{2C_{dot}} = \frac{e}{2C_{dot}} \quad (1)$$

With this voltage applied to quantum dot, an electron can tunnel through coulomb blockade of the quantum dot.

Working for single electron transistor in nutshell

A single electron transistor is shown in figure. As opposed to the semiconductor channel in a field effect transistor, the SET has an electrically isolated quantum dot located between the source and drain.

1. The SET is OFF mode. The corresponding potential energy diagram shows that it is not energetically favourable for electrons in the source to tunnel to the dot as shown in figure.
2. The SET is ON mode. At the lowest setting electrons tunnel one at a time, via the dot, from the source to the drain as shown in figure.
3. This is made possible by first applying the proper gate voltage, $V_{gate} = e/2C_{dot}$, so that the potential energy of the dot is made low enough to encourage an electron to tunnel through the coulomb blockade energy barrier to the quantum dot.
4. Once the electron is on it, the dots potential energy rises as shown in figure
5. The electron then tunnels through the coulomb blockade on the other side to reach the lower potential energy at the drain as shown in figure.
6. With the dot empty and the potential lower again the process repeats as shown in figure.

Advantages

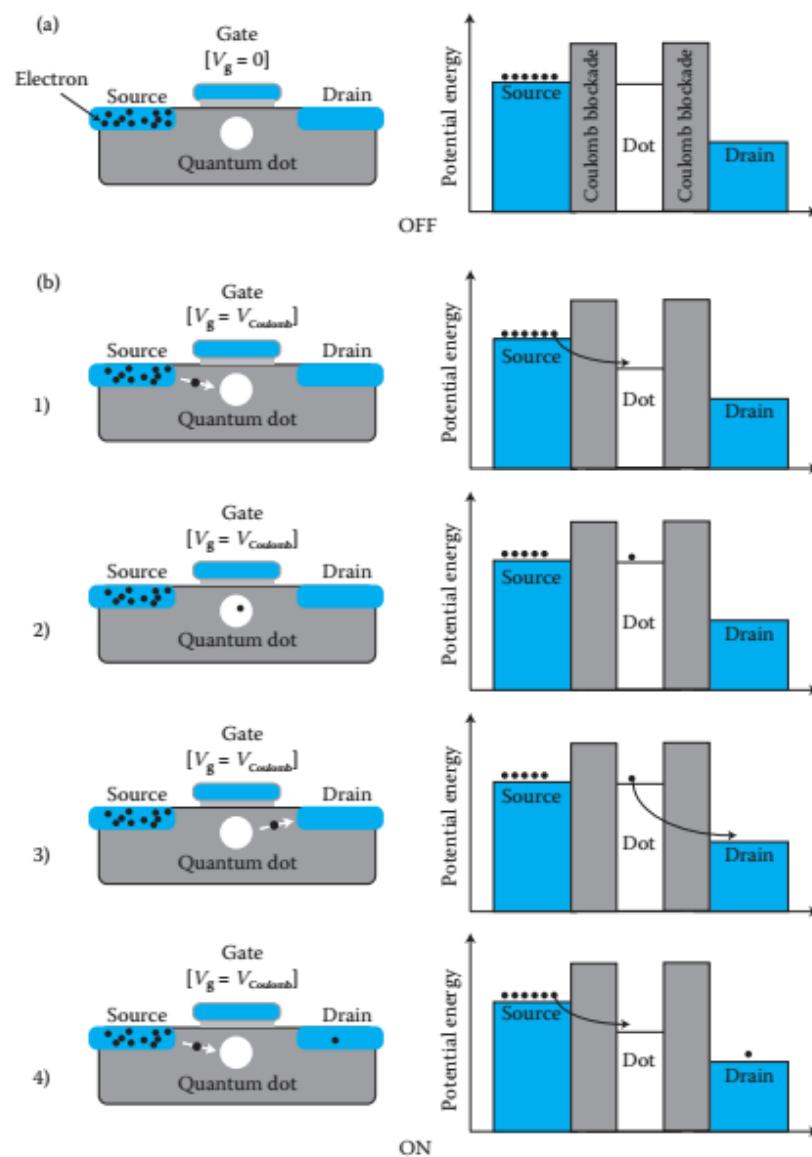
1. The fast information transfer velocity between cells is carried out via electrostatic interactions only.
2. No wire is needed between arrays. The size of each cell can be as small as 2.5nm. This made them suitable for high density memory.
3. This can be used for the next generation quantum computer.

Limitations

1. In order to operate SET circuit at room temperature, the size of the quantum dot should be smaller than 10nm
2. It is very hard to fabricate by traditional optical lithography and semiconductor processes
3. The method must be developed for connecting the individual structures into logic circuits and these circuits must be arranged into larger 2D patterns.

Applications

1. SET are used in sensor and digital electronic circuits
2. Variety of digital logic functions, including AND or NOR gates, is obtained based on SET operating at room temperature.
3. It is used for mass storage
4. It is used in highly sensitive electrometer.
5. SET can be used as a temperature probe, particularly in the range of very low temperatures.
6. SET is a suitable measurement setup for single electron spectroscopy.
7. It is used for the fabrication of homo-dyn receiver operating at frequencies between 10 and 300 MHz.



5.9. Quantum dot laser

Principle

A quantum dot laser is a semiconductor laser that uses quantum dots as the active medium in its light emitting region.

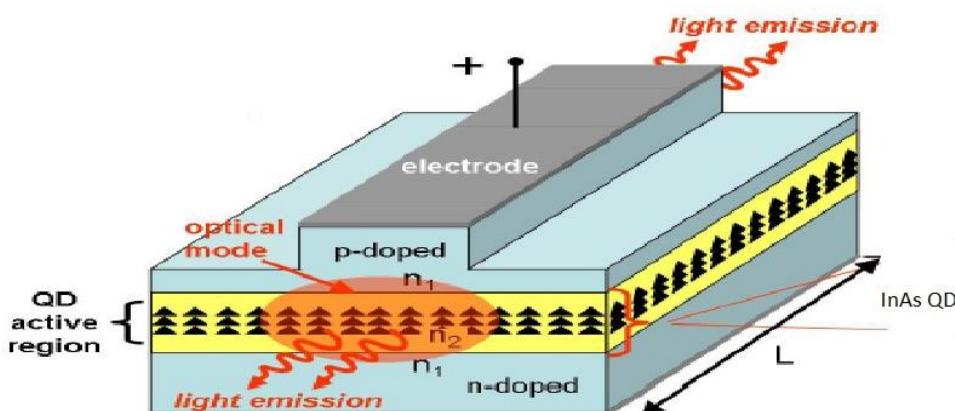
Construction

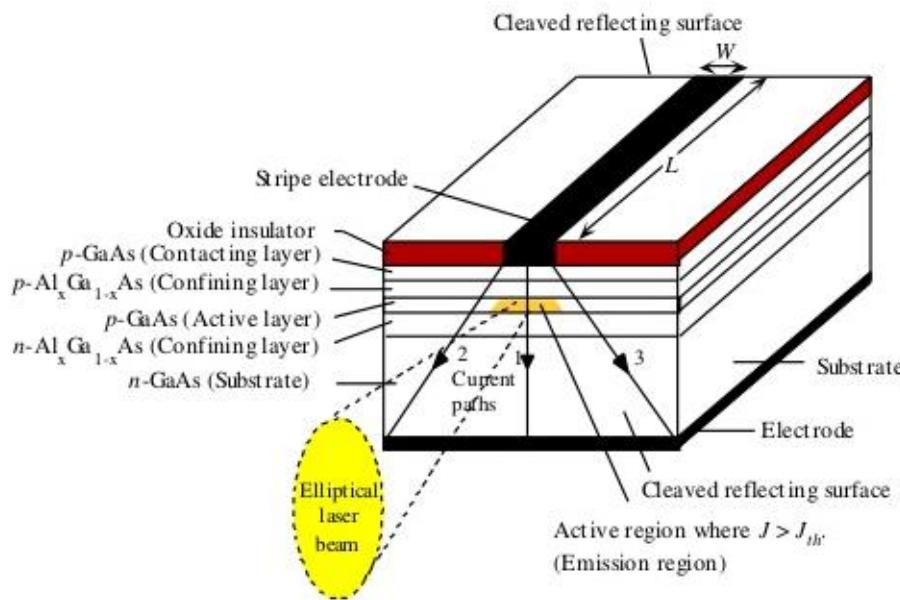
Figure shows a quantum dot near infrared laser diode grown on an n doped GaAs substrate. The top p metal layer has a GaAs contact layer. Immediately below it there are a pair of $2\mu\text{m}$ thick $\text{Al}_{0.85}\text{Ga}_{0.15}\text{As}$ cladding bounding layers that surrounds a 190 nm thick waveguide made of $\text{Al}_{0.05}\text{Ga}_{0.95}\text{As}$ in between p metal and n substrate. The front view of quantum laser diode is shown in figure. Here the waveguide plays the role of conducting the emitted light to the exit ports at the edges of the structure.

The waveguide is a 30 nm thick GaAs region, an centred in this region are 12 monolayers of $\text{In}_{0.5}\text{Ga}_{0.5}\text{As}$ quantum dots with a density of $1.5 \times 10^{10}\text{ cm}^2$. The details of the wavelength region is drawn below

Working

1. The electron and hole recombination causes the emission of laser light.
2. By varying the length L_c and width W the laser light with particular wavelength will be emitted.
3. A particular wavelength of $1.32\text{ }\mu\text{m}$ which is near infrared region can be produced for a current setting just above the 4.1 mA threshold value, labelled point a as shown in figure.
4. The faces of the layer were coated with high reflected material where the light is reflected back and forth to increase the stimulated emission and in turn the laser emission is enhanced.





5.10. Ballistic transport

Definition

When the mean free path of the electron is longer than the dimension of the medium through which the electron travels is called ballistic transport

Explanation

When the length L of the conductor becomes much smaller than the mean free path L_m the transport is termed *ballistic* meaning that the electrons do not scatter during the time it takes to travel through the conductor.

For example, ballistic transport can be observed in a metal nano wire. This is because the wire is of the size of a nanometer and the mean free path can be longer than in a metal.

Condition for ballistic transport

The mean free path can be increased by reducing the number of impurities in a crystal or by lowering its temperature.

i.e., $L \ll L_m$ and $L \ll L_\varphi$

where L length of the conductor, L_m mean free path, L_φ length over which an electron can travel before having an elastic collision. This is also called phase coherence length since it is the length over which an electron wave function retains its coherence. For $L \ll L_m$ and $L \ll L_\varphi$, we have ballistic transport. Ballistic transport occurs over very small length scales, and is obviously coherent.

The electron does not hit anything as it travels through the material and therefore there is no momentum or phase relaxation. Thus in ballistic material, the electron wave function can be obtained from schrodinger's equation.

Application

It is used in ultra-short channel semiconducting FETs or carbon nanotube transistors.

5.11. Quantum conductance and resistance

The quantum conductance is the quantised unit of electrical conductance denoted by G_0

$$G_0 = \frac{2e^2}{h} = 7.748 \times 10^{-5} \text{ mho}$$

The reciprocal of the quantum conductance is quantum resistance denoted by R_0

$$R_0 = \frac{h}{2e^2} = 12.9 \text{ k}\Omega$$

Derivation

A one dimensional quantum wire connects adiabatically two reservoirs with chemical potential μ_1 and μ_2 . The connections are assumed to be non-reflecting.

Reservoirs with chemical potential μ_1 and μ_2

It is also assumed that the wire is sufficiently narrow so that only the lowest transverse mode in the wire is below the fermi energy (E_F)

The current density is given by $J = -nev_d$ (1)

The density of electrons is determined by

$$n = \int_{\mu_2}^{\mu_1} dn = \int_{\mu_2}^{\mu_1} \frac{dn}{dE} dE = \frac{dn}{dE} \int_{\mu_2}^{\mu_1} dE$$

$$n = \frac{dn}{dE} [\mu_1 - \mu_2]$$

DR P R E S T C H E F A M I N O P Q R S T U V W X Y Z
(2)

Substituting equation (2) in (1), we get $J = -\frac{dn}{dE} [\mu_1 - \mu_2] eV_d$

DR P R E S T C H E F A M I N O P Q R S T U V W X Y Z
(3)

where dn/dE is the density of states.

We know that $E = N h v$ (4)

Where N is the number of electrons

We know that $n = \frac{N}{V} = \frac{\text{No.ofelectrons}}{\text{volume}} = \frac{\text{No.ofelectrons}}{\text{length} \times \text{area}} = \frac{N}{A \times l}$

Therefore $N = nAl$ (5)

Substituting equation (5) in (4). We get

$$E = n h v A l$$

Differentiating we get

$$dE = dn h v A l$$

$$\therefore \frac{dn}{dE} = \frac{1}{hvAl} \quad (6)$$

According to spin degeneracy, multiply equation (6) by two, we get

$$\frac{dn}{dE} = \frac{2}{hvAl} \quad (7)$$

If V is the voltage between two reservoirs, then we can write

$$\mu_1 - \mu_2 = -eV \quad (8)$$

Substituting equation (7) and (8) in equation (3) we get

$$J = \frac{2}{hvAl} eVeV_d$$

$$\text{(or)} \quad \frac{JvAl}{V_d} = \frac{2}{h} e^2 V$$

$$\text{(or)} \quad \frac{IvAl}{AV_d} = \frac{2}{h} e^2 V = \frac{Ivl}{V_d} = \frac{2}{h} e^2 V \quad (9)$$

We know that velocity = distance / time and frequency = 1/ time

$$\therefore V_d = \frac{l}{t} \quad (10)$$

$$\text{and} \quad \therefore V = \frac{1}{t} \quad (11)$$

Substitute (10) and (11) in (9), we get

$$\begin{aligned} \frac{Il}{t(l/t)} &= \frac{2}{h} e^2 V \\ \text{(or)} \quad \frac{I}{V} &= \frac{2e^2}{h} \\ \text{(or)} \quad \frac{1}{R_0} &= \frac{2e^2}{h} \end{aligned} \quad (12)$$

Therefore equation (13) can also be rewritten as

$$G_0 = \frac{1}{R_0} = \frac{2e^2}{h} \quad (13)$$

$$\text{Therefore quantum conductance } G_0 = \frac{1}{R_0} = \frac{2e^2}{h} = 7.748 \times 10^{-5} \text{ siemens (or) mho} \quad (14)$$

$$\text{And quantum resistance } R_0 = \frac{h}{2e^2} = 12.9k\Omega \quad (15)$$

Here G_0 is a fundamental unit

If there are N electronic channels, then equation (14) becomes

$$G_0 = \frac{2e^2}{h} N \text{ called Landauer formula} \quad (16)$$

i.e., $G = G_0 N$

$$\text{Similarly } R_0 = \frac{h}{2e^2} \times \frac{1}{N} = \frac{R_0}{N} \quad (17)$$

As the number of electronic channels increases, conductance increases and resistance decreases. The classical theory also predicts this behaviour, although the quantum theory shows that this happens in discrete steps, as the number of electron channel increases.

As N gets very large, the electron channels essentially form a continuum and the quantum theory tends towards the classical limit.

5.12. Metallic Nanowire

Consider a circular cross section wire which has a radius a and length L . Assume that L is very large relative to its mean free path.

Let us assume a copper wire having radius $a = 10 \text{ mm}$, $R = 5.395 \times 10^{-5} \text{ ohms / meter}$ and $\sigma = 5.9 \times 10^7 \text{ S/m}$. we need 18357m for 1Ω resistance to be maintained in the given radius of the wire. For $a = 10 \mu\text{m}$, $R = 53.05\Omega / \text{m}$ amounting to 1Ω in only 1.85 cm . If $a = 10 \text{ nm}$ then the resistance is huge.

Here the radius of the wire having radius on the order of its mean free path or less will have different conductance value compare to the bulk scale. For example, copper has a mean free path of approximately 40 nm and hence radius dependence effects usually occurs when the radius is double this value ($80 - 100 \text{ nm}$).

A $1 - 20\text{nm}$ radius range, the conductivity values changes from the bulk value and decreases as the radius a decreases. This is due to scattering effect from the wire surface, grain boundaries, defect free metals at small scales.

A relatively simple approximation for the resistivity of rectangular cross section of wires is

$$\rho = \rho_0 \left\{ \frac{1}{3 \left[\frac{1}{3} - \frac{\alpha}{2} + \alpha^2 - \alpha^3 \ln \left[1 + \frac{1}{\alpha} \right] \right]} + \frac{3}{8} C(1-P) \frac{1+AR}{AR} \times \frac{L_m}{W} \right\} \quad (1)$$

$$\text{Where } \alpha = \frac{L_m}{d} \times \frac{R_c}{1-R_c}$$

Where ρ_0 – bulk resistivity

W – wire width

AR – aspect ratio (wire height / wire width)

d – average grain size

P – specularity parameter

R_c – grain boundary reflection coefficient

C – constant

Here the first term is related to grain boundary scattering and the second term wire surface scattering. Both P and R_c can take values between 0 and 1. The experimental results are $P = 0.3 - 0.5$ and $R_c = 0.2 - 0.3$.

The proceeding model may work down to wire cross sectional dimensions on the order of 5 – 10 nanometres. Below which a quantum wire model that accounts for transverse quantization would be necessary. However as complicated as surface and grain boundary scattering are, other factors also determine the conductivity of nanowire

5.13. Carbon Nanotubes

The hexagonal lattice of carbon is simply graphite. A single layer of graphite is called graphene. CNT consists of a graphene layer rolled up into a cylindrical shape like a single molecule where each molecule nanotube is made up of a hexagonal network of covalently bonded carbon atoms Eg: fullerene. In some cases, the hexagon are arranged in a spiral form, the layer appears like a net having a large hexagonal mesh. The carbon nanotubes are hollow cylinders of extremely thin diameter, 10,000 times smaller than a human hair.

Structures of CNT

The CNTs have many structures on the basis of their length, type of spiral and number of layers. Their electrical properties depend on their structure and they act as either a metal or a semiconductor.

Types of CNT:

- (i) Arm chair
- (ii) Zigzag
- (iii) Chiral

- The axis of tube parallel to c-c bonds of the carbon hexagons are arm chair
- The axis of the tube is perpendicular to c-c are zigzag structure
- The axis of tube is inclined to c-c are chiral structure

Classification:

- (i) Single walled CNTs
- (ii) Multi walled CNTs

in Multiwalled CNTs more than one CNTs are coaxially arranged

Properties:

Electrical:

- (i) CNTs are metallic (or) semi conducting depending on diameter of chirality
- (ii) The energy gap of semiconducting chiral carbon nanotubes is inversely proportional to the diameter of tube.

- (iii) The energy gap also varies along the tube axis and reaches a minimum value at the tube ends. This is due to the presence of localised defects at the ends due to the extra energy states.
- (iv) In SWCNT conduction occurs through discrete electronic states that are coherent between the electrical contacts.

Mechanical:

- (i) The strength of C – C bond is very high leading to ultimate tensile strength
- (ii) Young's modules is 5 times greater than steel.
- (iii) Tensile strength is 50 times higher than steel
- (iv) Carbon nanotubes have ability to withstand extreme strength
- (v) It can recover from severe structural distortions due to rehybridization
- (vi) The strength of sp^2 C-C bond gives high hardness for CNTs

Physical

- (i) It have a high strength to weight ratio. This is indeed useful for light weight applications. (SWCNT $\rightarrow \rho = 0.8 \text{ g/cm}^3$; MWCNT $\rightarrow \rho = 1.8 \text{ g/cm}^3$).
- (ii) The surface are of nanotubes is of the order of $10-20 \text{ m}^2/\text{g}$ which is higher than that of graphite.

Chemical

- (1) They are highly resistant to any chemical reaction.it is difficult to oxidize them and the onset of oxidation in nanotubes is 100° C higher than that of carbon fibres.

Thermal

Nanotubes have a high thermal conductivity and the value increase with decrease in diameter

Applications:

- (i) It is used in development of flat panel displays
- (ii) It is used to design LEDs, FET and as switching devices
- (iii) It is used to produce battery, solar and fuel cells
- (iv) It is used as sensitive detector of various gases.
- (v) It is used as a catalyst for chemical reactions.
- (vi) It provides light weight shielding material for electromagnetic radiation
- (vii) It is used in nano scale electronic devices
- (viii) CNTs are used in drug delivery



Figure 1. Schematic representation of rolling graphene layer to create CNT¹⁵.