Anna University Solved Problems

Unit - 1: Electrical Properties of Materials

1. **The Following data are given for copper**
   (i) Density = 8.92 x 10^3 kg/m^3 (ii) Resistivity = 1.73 x 10^{-8} Ω.m
   (iii) Atomic weight = 63.5
   Calculate the mobility and the average time collision of electrons in copper obeying classical laws. (*DECEMBER 2001*)

Given data(s): (i) Density (ρ) = 8.92 x 10^3 kg/m^3
(ii) Conductivity (σ) = 1/ρ = 5.780 x 10^7 $\Omega^{-1}m^{-1}$
(iii) Avagardo number (A) = 6.023 x 10^{23} kg/mol
(iv) Atomic weight (Z) = 63.5
(v) Charge of electron (e) = 1.6 x 10^{-19}C
(vi) Mass of electron (m) = 9.11 x 10^{-31} kg.

Formula(s): (i) Number of electrons per unit volume $n = \frac{\text{Density} \times \text{Avagardo No.}}{\text{Atomic weight}}$ m^{-3}

(ii) WKT, $\sigma = \frac{ne^2\tau}{m}$ (or) $\tau = \frac{\sigma m}{ne^2}$ sec

(iii) Mobility $\mu = \frac{\sigma}{ne}$ $m^2V^{-1}S^{-1}$

Calculation(s): (i) $n = \frac{8.92\times10^3 \times 6.023\times10^{23}}{63.5} = 8.46\times10^{25}\,m^{-3}$

(ii) $\tau = \frac{5.78\times10^7 \times 9.11\times10^{-31}}{8.46\times10^{25}\times\left(1.6\times10^{-19}\right)^2} = 2.43\times10^{-11}\,\text{sec.}$

(iii) $\mu = \frac{5.78\times10^7}{8.46\times10^{25}\times1.6\times10^{-19}} = 4.27\,m^2V^{-1}S^{-1}$

Answer(s):

(i) No. of electron / volume (n) = 8.46 x 10^{25} m^{-3}
(ii) Relaxation time (τ) = 2.43 x 10^{-11} sec
(iii) Mobility (μ) = 4.27 m^2 V^{-1} S^{-1}.

2. **The thermal conductivity of copper at 300 *K* is 470 *$Wm^{-1}K^{-1}$*. Calculate the electrical conductivity of cooper if the Lorentz number is 2.45 x 10^{-8} WΩK^{-2}. (*JUNE 2010*).**

Given data(s): (i) Temperature (T) = 300 K
(ii) Thermal conductivity (K) = 470.4 $Wm^{-1}K^{-1}$
(iii) Lorentz Number (L) = 2.45 x 10^{-8} WΩK^{-2}.

Formula(s): (i) WKT, $\frac{K}{\sigma} = LT$ (or) $\sigma = \frac{K}{LT}$ $\Omega^{-1}m^{-1}$

Calculation(s): (i) $\sigma = \frac{470.4}{2.45\times10^{-8}\times300} = 6.4\times10^7\,\Omega^{-1}m^{-1}$

Answer(s):

(i) Electrical conductivity of copper at 300K is σ=6.4 x 10^7 $\Omega^{-1}m^{-1}$

3. Calculate the electrical conductivity in copper if the mean free path of electrons is 4 x 10^{-8}m, electron density is 8.4 x 10^{28} m^{-3} and the average thermal velocity of electron is 1.6 x 10^6 ms^{-1} (*DECEMBER 2012*)

Given data(s): (i) Mean free path of electron (λ) = 4 x 10^{-8} m
(ii) Electron density (n) = 8.4 x 10^{28} m^{-3}
(iii) Average thermal velocity of electron (v) = 1.6 x 10^6 ms^{-1}.
(iv) Charge of the electron (e) = 1.6 x 10^{-19} C
(v) Mass of the electron (m) = 9.11 x 10^{-31} kg.

Formula(s): (i) $\sigma = \frac{ne^2\lambda}{mv}$ $\Omega^{-1}m^{-1}$ $\quad \because \frac{\lambda}{v} = \tau$

Calculation(s): (i) $\sigma = \frac{8.4\times10^{28}\times\left(1.6\times10^{-19}\right)^2\times4\times10^{-8}}{9.11\times10^{-31}\times1.6\times10^{6}} = 5.9\times10^{7}\ \Omega^{-1}m^{-1}$

Answer(s):
(i) Electrical conductivity of copper is σ=5.9 x 10^7 $\Omega^{-1}m^{-1}$

4. Calculate the electrical and thermal conductivities for a metal with a relaxation time of 10^{-14} seconds. If the density of electrons is 6 x 10^{28} m^{-3}, calculate its Lorentz number using the above result. *(JUNE 2013).*

Given data(s): (i) Relaxation time (τ) = 10^{-14} sec
(ii) Temperature (T) = 300 K
(ii) Electron density (n) = 6 x 10^{28} m^{-3}
(iii) Mass of the electron (m) = 9.11 x 10^{-31} kg.
(iv) Charge of the electron (e) = 1.6 x 10^{-19} C
(v) Boltzmann constant (k) = 1.38 x 10^{-23} J K^{-1}.

Formula(s): (i) $\sigma = \frac{ne^2\tau}{m}$ $\Omega^{-1}m^{-1}$

(ii) $K = \frac{\pi^2}{3}\times\frac{nk^2\tau T}{m}$ Wm^{-1} K^{-1}

(iii) $L = \frac{K}{\sigma T}$ $W\Omega K^{-2}$

Calculation(s): (i) $\sigma = \frac{6\times10^{28}\times\left(1.6\times10^{-19}\right)^2\times10^{-14}}{9.11\times10^{-31}} = 1.686\times10^{7}\ \Omega^{-1}m^{-1}$

(ii) $K = \frac{\pi^2}{3}\times\frac{6\times10^{28}\times\left(1.38\times10^{-23}\right)^2\times10^{-14}\times300}{9.11\times10^{-31}} = 123.79\ Wm^{-1}K^{-1}$

(iii) $L = \frac{123.79}{1.686\times10^{7}\times300} = 2.45\times10^{-8}\ W\Omega K^{-2}$

Answer(s): (i) Electrical conductivity (σ) = 1.686 x 10^7 $\Omega^{-1}m^{-1}$
(ii) Thermal Conductivity (K) = 123.79 $Wm^{-1}K^{-1}$
(iii) Lorentz's Number (L) = 2.45 x $10^{-8}$$W\Omega K^{-2}$.

5. **Calculate the drift velocity and thermal velocity of conduction electrons in copper at a temperature of 300 K. When a copper wire of length of 2m and resistance 0.02Ω carries a current of 15 A. (Mobility (μ) = 4.3 x 10^{-3} m^2 V^{-1} S^{-1}** *(JAN 2014).*

Given data(s): (i) Temperature (T) = 300 K
(ii) Length of wire (L) = 2 m
(iii) Mass of the electron (m) = 9.11 x 10^{-31} kg.
(iv) Resistance (R) = 0.02 Ω
(v) Current (I) = 15 A
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(vi) Mobility (μ) = 4.3 x 10^{-3} m^2 V^{-1} S^{-1}

Formula(s): (i) V= I R volts
(ii) E = V / L volt m^{-1}
(iii) $v_d = \mu E$ m s^{-1}
(iv) $v = \sqrt{\frac{3kT}{m}}$ ms^{-1} (Remember equipartition energy eqn.)

Calculation(s): (i) V = 15 x 0.02 = 0.3 volts
(ii) E = 0.3 / 2 = 0.15 volt m^{-1}
(iii) v_d = 4.3 X 10^{-3} x 0.15 = 0.645 x 10^{-3} m s^{-1}.
(iv) $v = \sqrt{\frac{3\times1.38\times10^{-23}\times300}{9.11\times10^{-31}}} = 1.168\times10^{5}$ ms^{-1}

Answer(s):
(i) Drift velocity (v_d) = 0.645 x 10^{-3} ms^{-1}
(ii) Average velocity (v) = 1.168 x 10^{5} ms^{-1}.

6. **Calculate the drift velocity of the free electrons in a copper wire whose cross sectional area is 1.0 mm^2 when the wire carriers a current of 1A. Assume that each copper atom contributes one electron to the electron gas. Given *n* = 8.5 x 10^{28} m^{-3}** *(MAY 2016).*

Given data(s): (i) Conduction electron / m^3, n = 8.5 x 10^{28} m^{-3}
(ii) Charge of electron (e) = 1.6 x 10^{-19} C
(iii) Mass of the electron (m) = 9.11 x 10^{-31} kg.
(iv) Area of cross section (A) = 1.0 x 10^{-6} m^2
(v) Current (I) = 1 A
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Formula(s): (i) $v_d = \frac{I}{neA}$

Calculation(s): (i) $v_d = \frac{1}{8.5\times10^{28}\times1.6\times10^{-19}\times1\times10^{-6}} = 7.35\times10^{-5}$ ms^{-1}

Answer(s): Drift velocity (v_d) = 7.35 x 10^{-5} ms^{-1}.

7. A metallic wire has a resistivity of 1.42 x 10^{-8} Ωm. For an electric field of 0.14 V/m. Find (i) average drift velocity and (ii) mean collision time, assuming that there are 6 x 10^{28} electrons /m^3 *(APRIL 2015).*

Given data(s): (i) Conduction electron / m^3, n = 8.5 x 10^{28} m^{-3}
(ii) Charge of electron (e) = 1.6 x 10^{-19} C
(iii) Mass of the electron (m) = 9.11 x 10^{-31} kg.
(iv) Electric field (E) = 0.14 V m^{-1}
(v) Resistivity (ρ) = 1.42 x 10^{-8} Ω m

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Formula(s): (i) $\rho = \frac{m}{ne^2\tau}$ (or) $\tau = \frac{m}{ne^2\rho}$

(ii) $v_d = \left(\frac{eE\tau}{m}\right)$

Calculation(s): (i) $\tau = \frac{9.11\times10^{-31}}{6\times10^{28}\times\left(1.6\times10^{-19}\right)^2\times1.42\times10^{-8}} = 4.17\times10^{-14}$ sec.

(ii) $v_d = \left(\frac{1.6\times10^{-19}\times0.14\times4.17\times10^{-14}}{9.11\times10^{-31}}\right) = 1.025\times10^{-3}$ ms^{-1}

Answer(s): **(i)** Relaxation time (τ) = 4.17 x 10^{-14} sec.
(ii) Drift velocity (v_d) = 1.023 x 10^{-3} ms^{-1}.

8. If the energy level is lying 0.01 eV above Fermi level in a solid, what is the probability of this level being occupied by an electron at 270 K? *(JUNE 2010).*

Given data(s): (i) E – E_F = 0.01 eV
(ii) Boltzmann constant (K) = 1.38 x 10^{-23} J K^{-1}
(iii) Temperature (K) = 270 K

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Formula(s): (i) $F(E) = \frac{1}{1+e^{(E-E_F)/kT}}$

Calculation(s): (i) $F(E) = \frac{1}{1+e^{\left(0.01\times1.6\times10^{-19}\right)/(1.38\times10^{-23}\times270)}}$

$$F(E) = \frac{1}{1+e^{0.4294}}$$

$$F(E) = \frac{1}{1+1.5364} = 0.39$$

Answer(s): **(i)** F(E) = 0.39 (it is the probability of occupancy).

9. Evaluate the Fermi function for an energy kT above the Fermi energy? *(JUNE 2019).*

Given data(s): (i) $E - E_F = kT$

(ii) Boltzmann constant (K) = 1.38×10^{-23} J K $^{-1}$

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Formula(s): (i) $$F(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

Calculation(s): (i) $$F(E) = \frac{1}{1+e^{(kT)/(kT)}}$$

$$F(E) = \frac{1}{1+e^{1}}$$

$$F(E) = \frac{1}{1+2.7138} = 0.2689$$

Answer(s): **(i)** F(E) = 0.2689(it is the probability of occupancy).

10. Calculate the Fermi energy of copper at 0 K if the concentration of electron is 8.5 x 10^{28} m^{-3}? *(APRIL 2017).*

Given data(s): (i) $n_c = 8.5 \times 10^{28}$ m^{-3}

(ii) h = 6.6.26 x 10^{-34} J s^{-1}

(iii) m = 9.11×10^{-31} kg.

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Formula(s): (i) $$E_F = \left(\frac{3n_c}{8\pi}\right)^{2/3} \times \frac{h^2}{2m}$$

Calculation(s): (i) $$E_F = \left(\frac{3\times 8.5\times 10^{28}}{8\pi}\right)^{2/3} \times \frac{\left(6.626\times 10^{-34}\right)^2}{2\times 9.11\times 10^{-31}}$$

$$E_F = \left(1.0146\times 10^{28}\right)^{2/3} \times 2.40965\times 10^{-37}$$

$$E_F = 1.12932\times 10^{-18}\ \text{J}$$

$$E_F = \frac{1.12932\times 10^{-18}}{1.6\times 10^{-19}} = 7.05\,\text{eV}$$

Answer(s): **(i)** Fermi energy at T= 0 K is E_F = 7.05 eV.

11. Calculate the temperature at which there is 1% probability of a state with energy 0.5 eV above Fermi energy. *(JUNE 2011).*

Given data(s): (i) F(E) = 1% = 0.01

(ii) k = 1.38×10^{-23} J K^{-1}

(iii) $E - E_F$ = 0.5 eV.

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Formula(s): (i) $$F(E)=\frac{1}{1+e^{(E-E_F)/kT}}$$

Calculation(s): (i) $$\frac{1}{F(E)}=1+e^{(E-E_F)/kT}$$

$$\frac{1}{F(E)}-1=e^{(E-E_F)/kT}$$

Taking log on both sides

$$\log\left(\frac{1}{F(E)}-1\right)=\frac{E-E_F}{kT}$$

$$T=\frac{E-E_F}{k\left(\log\left(\frac{1}{F(E)}-1\right)\right)}$$

$$T=\frac{0.5\times1.6\times10^{-19}}{1.38\times10^{-23}\left(\log_e\left(\frac{1}{0.01}-1\right)\right)}$$

$$T=\frac{8\times10^{-20}}{1.38\times10^{-23}\times\left(\log_e 99\right)}$$

$$T=1261.58\,\text{K}$$

Answer(s): **(i)** Temperature (T) = 1261.58 K.

12. Calculate the carrier concentration of electrons in an energy interval of 0.01 eV above the Fermi level of sodium metal. The Fermi energy of sodium at 0 K is 3 eV. *(JUNE 2012).*

Given data(s): (i) E_F = 3 eV

(ii) h = 6.6.26 x 10^{-34} J s^{-1}

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Formula(s): (i) $$n=\frac{4\pi}{h^3}(2m)^{3/2}\times\int_{E_1}^{E_2}E^{1/2}dE$$

Calculation(s): (i) Fermi energy at T = 0 K, E = E_F = 3 eV

Therefore, E_1 = 3 x 1.6 x 10^{-19} = 4.8 x 10^{-19} J

(ii) Fermi energy above 0.01 eV is E_2 = E_1 + 0.01 eV

E_2 = 3 + 0.01 eV

E_2 = 3.01 eV

E_2 = 4.816 x 10^{-19} J

The Number of states per unit volume in the energy states between E_1 and E_2 are

$$n = \frac{4\pi}{h^3}(2m)^{3/2} \times \int_{E_1}^{E_2} E^{1/2} dE$$

(or) $n = \frac{4\pi}{h^3}(2m)^{3/2} \times \frac{2}{3}\left[E_2^{3/2} - E_1^{3/2}\right]$

(or) $n = \frac{4\pi}{\left(6.626\times10^{-34}\right)^3}\left(2\times9.11\times10^{-31}\right)^{3/2} \times \frac{2}{3}\left[\left(4.816\times10^{-19}\right)^{3/2} - \left(4.8\times10^{-19}\right)^{3/2}\right]$

(or) $n = \frac{6.181052\times10^{-44}}{\left(6.626\times10^{-34}\right)^3} \times \frac{1}{3}\left[\left(4.816\times10^{-19}\right)^{3/2} - \left(4.8\times10^{-19}\right)^{3/2}\right]$

(or) $n = \frac{6.181052\times10^{-44} \times 1.664154\times10^{-30}}{3\times\left(6.626\times10^{-34}\right)^3}$

(or) n = 1.1786 x 10^{26} m^{-3}

Answer(s): **(i)** Carrier concentration (n) = 1.1786 x 10^{26} m^{-3}