Problems:

1) Calculate the moment of inertia of a solid cylinder of mass 200gms and diameter 10cm with respect to its own axis

Given data: (M) = 200gms (D) = 10cm For: $I = MR^2$ Ans: $I = 5 \times 10^{-4}$ kgm^2

2) A solid sphere of mass 2kg rotates with respect to its diameter with angular speed 5 rotation / second. The radius of the sphere is 25cm. What is the (i) Moment of inertia & (ii) Kinetic energy associated with the rotation of the sphere with respect to its diameter?

Given: M = 2 kg; $\omega = 5 \times 2\pi$ rad/sec; R = 25cm For: $I = MR^2$; $E = \frac{1}{2} I\omega^2$

$I = 0.1250$ kgm^2 / $E = 61.62$ J.

3) Two bodies of masses 6kg and 10kg have position (along the x-axis) 5m and 10m respectively from the origin. Find the distance of the centre of mass from the origin and moment of inertia with respect to the axis of rotation through its centre of mass.

Given: $M_1 = 6$kg; $M_2 = 10$kg Position of M_1 at $x_1 = 5$m; M_2 at $x_2 = 10$m

C.M = ? I = ?

$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = 8.125$m $I = m_1 x^2 + m_2 x^2 = 93.76$ kgm^2

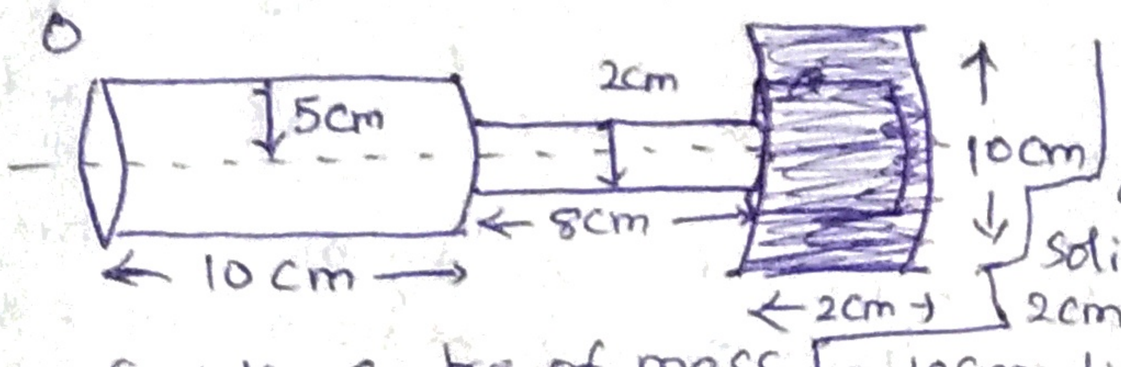
(or) $I = \mu x^2$

4) Two particles of masses 3kg & 5kg are seperated by a distance 4m along x-axis. Locate the position of centre of mass of the system and find out the moment of inertia

Given: $M_1 = 3$kg; $M_2 = 5$kg; $x = 4$m ($x = x_1 + x_2$); $M_1 x_1 = M_2 x_2$ ($x_2 = 1.5$; $x_1 = 2.5$)

$I = \mu x^2 = 30$ kgm^2

5)



A machine part is shown in figure. It consists of a 10cm long and 10cm diameter solid cylinder attached at one end of a 8cm long, 2cm diameter cylindrical rod. At other end of the rod a 10cm diameter and 2cm wide disc is attached. Find the COM of the machine part assuming it to be made of the homogeneous material.

find the centre of mass

cylinder 1 - $l = 10$cm; $R = 5$cm

cylinder 2 $l = 8$cm; $R = 1$cm

cylinder 3 $l = 2$cm; $R = 5$cm

$\rho = \frac{m}{A}$ or $m = \rho l$

$\therefore m_1 = \pi r^2 l \rho = \pi (5)^2 \times 10\rho = 250\pi\rho$

$m_2 = \pi r^2 l \rho = \pi (1)^2 \times 8 = 8\pi\rho$

$m_3 = \pi \times 25 \times 2\rho = 50\pi\rho$

$x_1 = 5$cm; $x_2 = 14$; $x_3 = 19$

$\therefore x_{cm} = \frac{250\pi\rho \times 5 + 8\pi\rho \times 14 + 50\pi\rho 19}{250\pi\rho + 8\pi\rho + 50\pi\rho}$

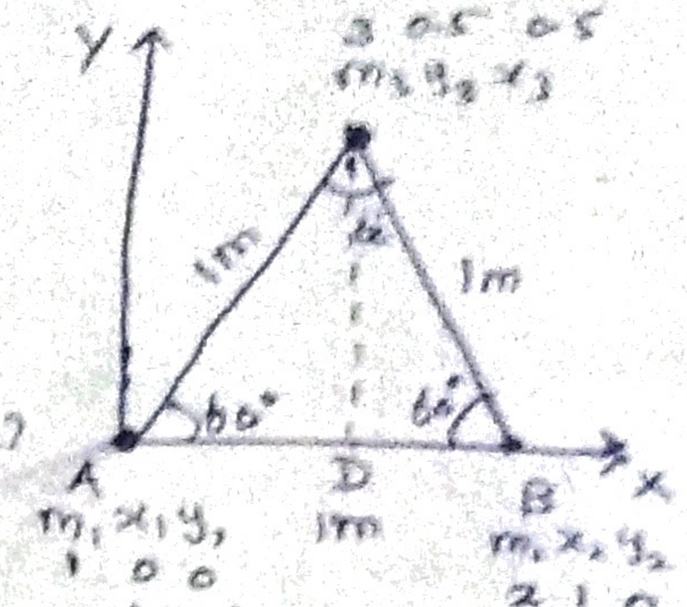
$= \frac{(1250 + 112 + 950)\pi\rho}{308\pi\rho} = \frac{2312}{308} = 7.5$m

1250
112
950
2312

6) Locate the centre of mass of a system of particles of masses $m_1 = 1kg$, $m_2 = 2kg$ and $m_3 = 3kg$, situated at the corners of an equilateral triangle of side 1cm.

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} \quad ; \quad y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

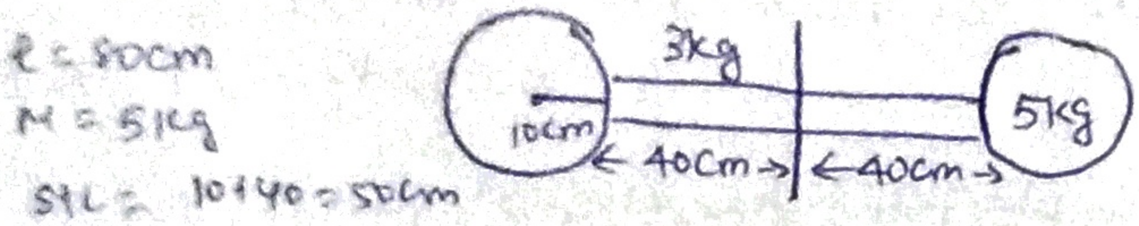
COM - 3.5/6 , $\sqrt{3}/4$



7) The radius of gyration of a disc about its centre of mass is 1.41m. If the mass of the disc 1kg, what is its moment of inertia?

$I = mk^2$ $M = 1kg$; $K = 1.41m$ $I = 2kg-m^2$

8) Find the moment of inertia about the geometric centre of the given structure made up of one thin rod connecting two similar solid spheres as shown in figure



R = 50cm
M = 5kg
SL = 10+40 = 50cm
R = 10cm

$I_{rod} = \frac{ML^2}{12}$ $I_C = \frac{2}{5}MR^2$

$I_{SP'} = I_C + MR^2_{(S+L)}$

$I_T = I_{rod} + I_{SP} = 2.7 kgm^2$

9) A wheel in the form of uniform disc of diameter 1m & mass 100kg is rolling on a horizontal plane with a speed of 5 m/sec. Find the K.E of the wheel.

Data: $M = 100kg$; $R = 0.5m$; $v = 5 m/s$ $E = \frac{1}{2}mv^2 = 1250J$ $E_K = \frac{1}{2}I\omega^2$ $I = \frac{MR^2}{2}$

R K.E $E_K = 625$; T K.E $E_T = 1125J$ $\omega^2 = \left(\frac{v}{r}\right)^2 = \left(\frac{5}{0.5}\right)^2 = 100 rad/s$ $= 12.5 kgm^2$

10) The angular momentum of rotation of a body is 75.36 Js & its rate of rotation is 24 rps. Calculate its momentum of inertia.

Data: $L = 75.36 Js$; $n = 24$; $\omega = 2\pi n = 48\pi$; $L = I\omega$ (or) $I = \frac{L}{\omega} = 0.5 kg\,m^2$

11) A fly wheel of moment of inertia $0.3 kg\,m^3$ is rotating at a rate of 2000rpm. what torque is needed to bring it at rest in 10s?

Data: $I = 0.3$; $n = 200 rpm$; $t = 10s$ $\omega_0 = \frac{2\pi n}{t} = \frac{2\pi \times 200}{60} = 20.93 rad/s$

$\omega = 0$, $t = 10s$ $\alpha = \frac{\omega - \omega_0}{t} = -\frac{20.93}{10} = -2.093 rad\,s^{-1}$ $\tau = I\alpha \Rightarrow 0.63 N-m$

12) Consider the earth as a uniform sphere of mass $5.98 \times 10^{24} kg$ and radius $6.37 \times 10^6 m$ spinning on its axis at the rate of per day. Find the angular momentum associated with this spinning

Data: $M = 5.98 \times 10^{24} kg$; $R = 6.37 \times 10^6 m$; $L = I\omega$; $I = \frac{2}{5}MR^2$; $\omega = \frac{2\pi}{60 \times 60 \times 24}$; $L = 7.055 \times 10^{33}$ Nms (TL)