Velammal College of Engineering & Technology (Autonomous) Madurai 625 009
Department of Physics

PH25101 PHYSICS FOR MECHANICAL ENGINEERING

UNIT – II EFFECT OF FORCE

Introduction to Newtonian mechanics – Newton's law - Vector representation of Force - Parallelogram law for Addition of Forces – Lami's theorem - Triangular law of forces – Dot product & Cross product - classification of the system of forces – Problems on forces using vector representations.

2. Introduction

For many years the cause of motion of a body was nor clear and remained the central theme of natural philosophy. It was 1686 when a great English Physicist Sir Issac Newton summarized the basic principles which govern the motion of a body in the form of three laws.

2.1 . Newton's first law

Everybody continues in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise.

Example 1:

Let us consider that a coin of one rupee is placed on a paper and placed over the mouth of a bottle as shown in figure, If the paper is removed quickly by moving in the horizontal direction, then we can find the coin drops in the bottle.

Example 2:

If we take a rough block and keep it in a horizontal plane and move it with the help of a hand. As soon as we remove the force applied by our hand, we find that it stops very soon. Now if we put some lubricant on the surface and repeat the experiment, we observed that the block will move to a greater distance compared to the first case. Further, if we make the block and the plane smoother and apply lubricant, it moves slower and reaches more distance before coming to rest. Hence, it is concluded that, even if the friction is eliminated, the body will keep on moving with constant velocity.

2.1.1. Inertia

The tendency of the body to remain in its state of rest or of motion is because a body by itself is unable to change its position and this property is called inertia.

2.1.2. Momentum

A body's momentum is defined as the quantity of motion it possesses and is measured as the product of its mass and velocity.

2.1.3. Newton's second law

The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts. *When a body is acted upon by a force its resulting acceleration is directly proportional to the mass.*

Example

Consider two identical trolleys at rest on two adjacent horizontal railways as shown in figure. Let trolley A be pushed with a force F and trolley B with a force 2F simultaneously. We find that trolley B moves faster than trolley A. if we repeat the experiment several times by changing impressed force each time, we find that the greater the force, the faster the trolley moves, or the ratio of force and the acceleration acquired by the body each time is constant. "Thus the rate of change of momentum is directly proportional to the impressed force.

Now consider another set of two trolleys, one being filled A and the other one empty B. If these trolleys are pushed with the same force at the same time, we find that empty trolley A moves faster than as compared to filled one B. If we repeat this experiment several times by changing the mass of the trolley by removing some material from the trolley each time, we find that the lighter the trolley the faster its moves or greater the mass, the lesser will be change in velocity of the trolley. "Thus acceleration produced by a force in a body is inversely proportional to the mass of the body.

Hence we can write F = m x a

Let a body of mass *m* moving with a velocity *u* subjected to force *F*, change its velocity from *u* to *v* in *t* seconds. This means that because of the application of the force *F* on the body of mass *m* its momentum is changed from *mu* to *mv* in *t* seconds. Thus the rate of change of momentum of the body will be $\frac{mv-mu}{t}$

By Newton's second law of motion, the rate of change of momentum is directly proportional to applied force.

i.e., $F \propto \frac{mv-mu}{t}$ (or) $F \propto \frac{m(v-u)}{t}$ where *v* – *u* is the change in velocity hence the change in velocity per unit time is acceleration.

Here, the unit of force is different in S.I and CGS units.

For S.I. Unit, the unit of force is one Newton, which is defined as the force which produces an acceleration of 1 m/s^2 in a body of mass 1 kg.

In CGS unit, the unit of force is dyne. One dyne of a force is defined as a force capable of producing an acceleration of $1 cm/s^2$ when applied on a body of mass 1g. Here 1N = 10^5 dynes.

2.1.4. Mass and Weight

The mass of the body is defined as the quantity of matter contained in a body and is a measure of the inertia of the body.

We know that the body can be attracted to the centre of earth. If a mass is allowed to fall freely it will be attracted towards the centre of earth by a force which will produces acceleration in that body. The force with which a body is attracted towards the centre of earth is called gravitational attraction force

Weight of the body is the gravitational force with which earth attracts the body towards its centre

If a body of mass *m* is allowed to fall, it will be attracted by the earth with an acceleration of *g*, Then from the second law of motion, W = m g

Hence the weight of the body is *g* times the mass of the body and it is expressed in N (MKS system) and dyne (CGS system)

2.1.5. Newton's third law

It states that "*for every action there is always an equal and opposite reaction.*

Example

Motion of the lift

Weight of the body is defined as the force with which a body is attracted towards the centre of earth. When a body rests on a surface/platform which is accelerated, the weight of the body appears changed and the changed weight is called apparent weight and it depends on the acceleration of the platform. Let us consider a weight W = mg resting on the platform of a lift.

Case 1: Lift is unaccelerated

When lift is stationary *v* = 0 or moving with constant velocity. In this case, there is no reaction and the apparent weight *W* is equal to actual weight *W* (=*mg*)

i.e., W = W' = mg

Case 2: Lift is accelerated upward

If the lift is accelerated with a constant acceleration *a*, the net force acting on the man are its weight acting downward and the reaction (=ma) acting downward

∴ Apparent weight W' = W + R = m g + m a = m (g + a)

Hence, Apparent weight is > Actual weight

Case 3: Lift is accelerated downward a < g

If the lift is accelerated with a constant acceleration *a*, such that *a* < *g*, then the *reaction R = m a* will act upward while weight of the man W (=mg) will act downward, hence

W' = W – R = m g – m a (or) W' = m (g-a)

Case 4: Lift is accelerate downward with a > g

If the lift is accelerated with acceleration more than *g* the apparent weight will be

W = m (g – a) = -m(a-g)

Hence, under this situation the man will be accelerated upward and will stick to the ceiling of the lift. Here, reaction *ma* will be more than *mg*.

2.2.1 Addition of two forces

If we have to obtain the sum of two vectors A and B showing two successive displacements of a point. Their vector sum C is obtained by representing graphically vector A represent graphically vector B, then the line joining the head of B to the tail of A gives the vector sum of

A and B represented by C and direction is from the tail of first vector to the head of last vector. This law of vector addition is called triangle law of vector addition

If the two adjacent sides of a parallelogram represent the two given vectors A and B in direction as well as in magnitude, then the diagonal of the parallelogram will represent the vector sum C of the given two given vectors A and B. This law is called parallelogram law of vector addition.

2.2.2. Parallelogram law of forces

If a body is acted upon by two forces, then the magnitude and direction of the resultant force can be found with the help of parallelogram law of forces which states that "*If a particle is subjected to two forces represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, the resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through the same point.*

Two concurrent forces P and Q acting at a point O on a body are represented in figure by OA and OB, the resultant of these two forces can be found by the following construction: Complete the parallelogram OABC where AC is parallel to OB and BC is parallel to OA. Since OA = BC the forces P and Q may also represented by the sides OB and BC. Thus the line OC which is the resultant of OB and BC forces, will be the resultant R of two forces P and Q and can be found graphically. We can also calculate the value of resultant mathematically and to do so draw from C a normal CD on extended part of OB.

From right angle triangle BDC, we have CD = BC sin θ and BD = BC cos θ

Since ∠ ODC is of 90º. We have from the right angle triangle ODC, OC being hypotenuse

$OC^2 = OD^2 + CD^2$

(or) $OC^2 = (OB + BD)^2 + CD^2$ (as OD = OB + BD)

(or) $OC^2 = OB^2 + 2OB.\,BD + BD^2 + CD^2$

Substituting BD = BC cos θ and CD = BC sin θ,

We have $OC^2 = OB^2 + 2.\,OB.\,BC.\cos\theta + BC^2\cos^2\theta + BC^2\sin^2\theta$

(or) $OC^2 = OB^2 + 2.\,OB.\,BC.\cos\theta + BC^2(\cos^2\theta + \sin^2\theta)$

(or) $OC^2 = OB^2 + 2.\,OB.\,BC.\cos\theta + BC^2$

(or) $OC = \sqrt{OB^2 + 2.OB.BC.\cos\theta + BC^2}$

As lengths OC, OB and BC are perpendicular to R (resultant) Q and P respectively.

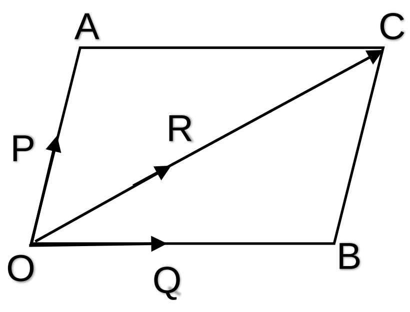
$\therefore R = \sqrt{Q^2 + 2.QP.\cos\theta + P^2}$

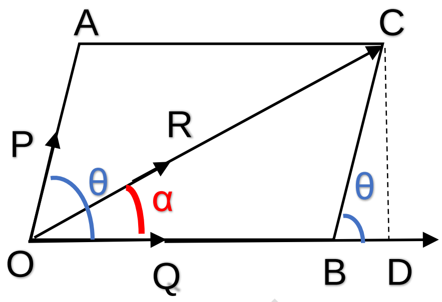
Let us assume that the resultant makes an angle α with a force Q. Then we have from the right angle triangle ODC,

$tan\alpha = \frac{CD}{OD} = \frac{CD}{OB+BD}$ (or) $tan\alpha = \frac{BC\,sin\theta}{OB+BC\,cos\theta}$

$$\therefore tan\alpha = \frac{P\,sin\theta}{Q + P\,cos\theta}$$

-



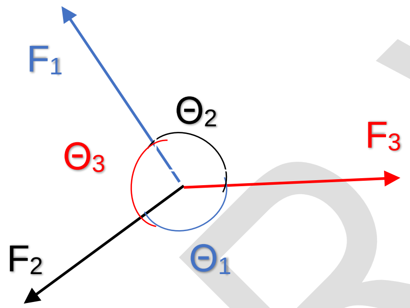


2.2.3. Lami's theorem

Consider three forces (F_1, F_2 and F_3) acting at a point as shown in figure. Let the angle between F_2 and F_3 be θ_1, between F_1 and F_3 be θ_2 and between F_1 and F_2 be θ_3. If the forces are in equilibrium, then according to Lami's theorem

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

Lami's theorem states that "if a body is in equilibrium under the action of three coplanar and concurrent forces, each of the force is proportional to the sine of the angle between the other two".

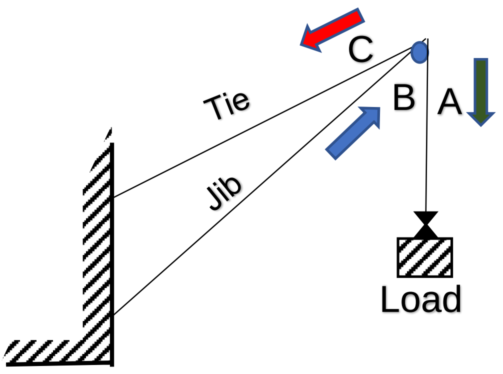


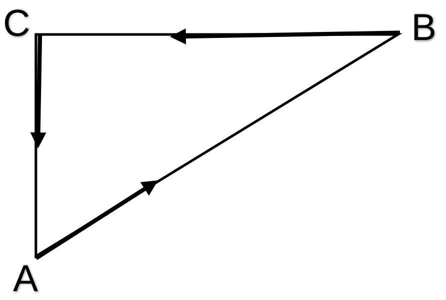
2.2.4. Triangular law of forces

If a body is subjected to three coplanar forces (coplanar forces are the forces whose line of action lies in the same plane), the conditions of the body being in equilibrium are:

(a) The force must be such that their line of action passes through a common point.
(b) It should be possible to represent the three forces by the three sides, of triangle, taken in order, each side of which is parallel to the line of action of the force.

The first condition is known as the principles of concurrence and the second condition gives the triangle of forces or triangle law of forces which can stated as: "if three forces acting on a point are in equilibrium, they can be represented by the sides of a triangle drawn parallel to the forces." It can also state as "If two forces acting upon a particle simultaneously are represented, both in magnitude and direction by the two sides of the triangle, taken in order the resultant will be represented, both in magnitude and direction by the third side of the triangle taken in opposite order.





This law is very useful in engineering problems especially when we know one force and we have to find the magnitude of the remaining two forces. This is illustrated by the following example: Figure shows an arrangement of Jib crane. The force acting are **A** load, **B**, the thrust of the jib and **C** the tension in the tie. In this case, we know the magnitude and direction of force **A** but we know only the direction of forces **B** and **C**. By constructing the triangle of forces as described above, we can find the value of **B** and **C** forces.

Equilibrium: A body is said to be in equilibrium when it is completely at rest.

Equilibrant: The equilibrant of a number of forces is that force which must be added to the forces in order to produce equilibrium. The equilibrant is always equal in magnitude to the resultant but opposite in direction.

2.2.5. Scalar product or Dot product

The scalar product of two vectors is defined as the product of the magnitude of the two vectors and the cosine of the smaller angle between them. It is represented by a dot between the vectors so it is also called dot product. We can write:

$A \cdot B = A.B \cos\theta = S$

Where S is the scalar quantity.

Also $A. B = A (B \cos\theta)$

$= A.$ (projection of B along A)

$= B (A \cos\theta)$

$= B.$ (projection of A along B)

From above we can also define scalar product of two vectors as the product of one vector with the projection of other vector along the direction of first.

2.2.5.1. Properties of scalar product

(1) Scalar product is commutative: since $\cos(-\theta) = \cos\theta$ we can say that A. B = B. A and hence scalar product is commutative.

(2) Scalar product is distributive:

$A.(B + C) = A\,|B + C| \cos\theta$

= A [projection of (B+C) along A]

= A (projection of B along A + projection of C along A)

= A. B + A. C which shows that scalar product is distributive

2.2.5.2. Cross (or) vector product of vectors

The vector product of two vectors is defined as a vector having a magnitude equal to the product of the magnitude of the two vectors and the sine of the angle between them and is in the direction perpendicular to the plane containing the two vectors.

i.e., C = A X B = A.B sin(A.B) n

where A and B are the magnitudes of the vectors. A and B, (A. B) is the angle between the two vectors and *n* is the unit vector perpendicular to the plane of vector A and B.

The direction of A X B will be taken positive or negative in accordance with right hand screw rule. This rule states that "when a screw is pointing upwards, then the direction along the screw C depends on the vector product of first vector A to the second vector B. If the screw is placed is placed downwards, the direction reversed.

The properties of vector product are as follows:

(1) A X B = = B X A (vector product is non-commutative)
(2) A X (B +C) = A X B + A X C (vector product is distributive)
(3) A X B = A B sin (A, B) n = A B sin 90° n = AB n
 The magnitude of the vector product of two vectors mutually perpendicular is equal to the product of the magnitude of the vectors.
(4) The vector product of two parallel vectors is zero or null vector.
 A X B = A B sin (A, B)n = A B sin 0° n = 0
(5) The vector product of a vector by itself is a null vector. (A X A = 0)

Vector product in terms of components

i x i = j x j = k x k = 0

i x j = n if the unit vector is perpendicular to both I and j along z axes.

i x j = k ; j x i = -k

j x k = i ; k x j = -i

k x i = j; k x i = - j

here A X B = $\begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

1. Ordinary triple product A (B.C) ≠ (A.B) C {here A is a vector and B,C are scalar)
2. Scalar triple product A. (B X C). Here a dot product of two vectors is a scalar and it is equal to the magnitude of the volume of the parallelopiped formed by three vectors A, B, and C.

Note: (A.B) X C is not possible. The cross product of a vector with a dot product of two vectors is undefined.

3. Vector triple product: A X (B X C) : here A X (B X C) must lie in the plane of B and C and perpendicular to the planes of B and C and perpendicular to C and
A X (B X C) ≠ (A X B) X C

2.3. system of forces

Force is the one which changes or tends to change the state of rest or of uniform motion of a body upon which it acts. A force represents the action of one body on another. Force is a vector quantity.

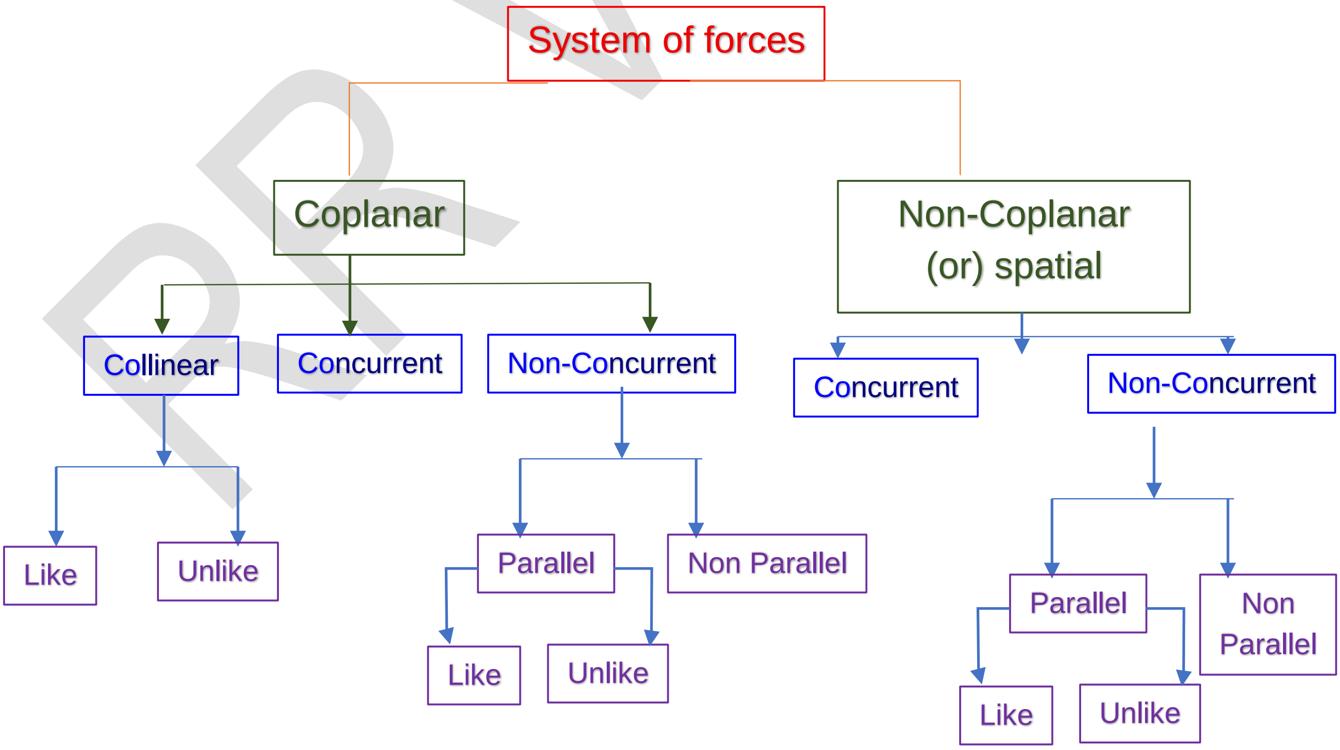
A force is characterized by

1. Magnitude
2. Line of action
3. Direction

The magnitude of the force is denoted in terms of Newton. An infinite straight line along which the force acts is called a line of action of the force. It is denoted by an angle with some fixed axis. This angle with the fixed axis and the sense of force represents the Direction of the force. Arrow head (sense) of the force indicates whether the force acts outwards from a particle or towards a particle.

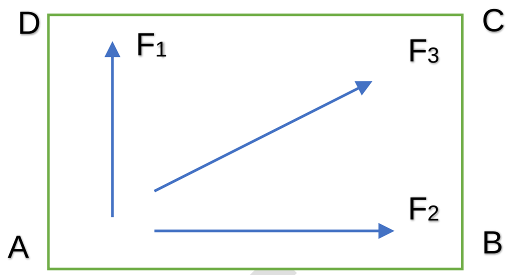
2.4. system of forces

A body with two or more forces acting simultaneously on it constitute a system of forces.



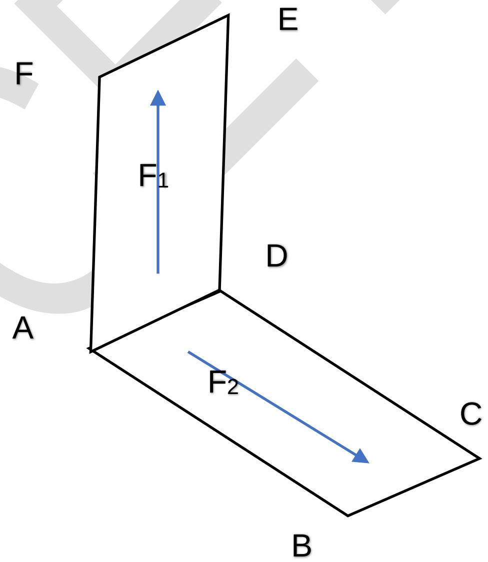
Coplanar Forces:

In coplanar force system, all the forces act in one plane. Here, the line of action of forces F_1, F_2 and F_3 lies in straight line ABCD. This system is also called as 'Forces in plane'.



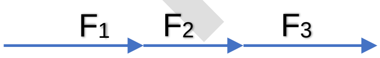
Non-coplanar forces

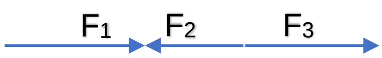
In a non-coplanar force system, the force do not act on one plane. In this figure the line of action of force F_2 lies in the ABCD plane.



Collinear Forces

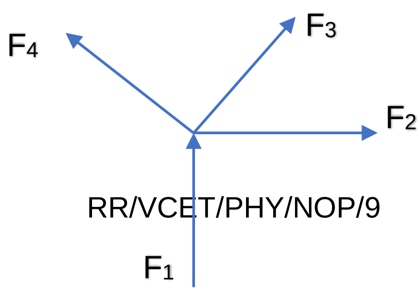
The forces which acts on a common line of action are called collinear forces. If they act in same direction, they are called like collinear an if they act in opposite direction, they are called 'unlike collinear'



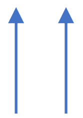


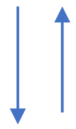
Concurrent Forces

In concurrent force system, the forces intersect at a common point as shown in figure

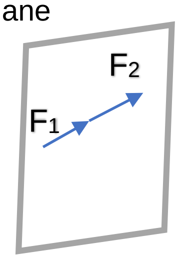


Parallel Forces: In parallel force system, the line of action of forces are parallel to each other. Parallel forces actin in the same direction are called like parallel forces and the parallel forces actin in opposite direction are called unlike parallel forces.

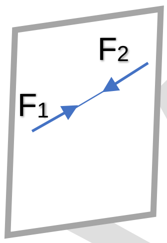




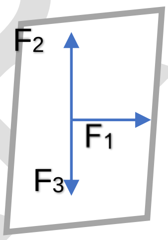
Like collinear coplanar forces: Forces acting in same direction, lies on a common line of action and acts in a single plane



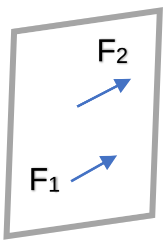
Unlike collinear coplanar forces: Forces acting in opposite direction , lies on a common line of action and acts in a single plane.



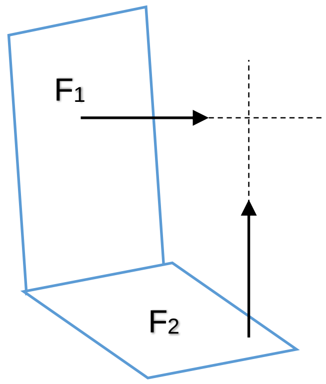
Coplanar concurrent forces: Forces intersects at a common point and lies in a single plane.



Coplanar non-concurrent forces: Forces which do not intersect at a common point, but acts in a single plane



Non-coplanar concurrent forces: Forces intersect at a common point, but their line of action do not lie on the same plane



Non-coplanar non-concurrent forces: Forces do not intersects at one point and also their line of action do not lie on the same plane