3. STATICS OF PARTICLES

Forces on a particle - resolution of a force- Resultant of several concurrent forces: Equilibrium of a particle - forces in space - equilibrium of a particle in space.

3.1. Introduction

A particle may be defined as a portion of matter that is infinitely small in all directions. i.e., a particle has no size, but it has mass. It is noted that the term particle is only a relative term. For example, in astronomical calculations, the earth may be assumed to be a particle; but to an earth-bound observer, the earth is a body of great size. For mathematical description, a particle denotes a body in which all the materials are concentrated at a point. As all the materials are concentrated at a point, a particle is always subjected to a coplanar collinear (or) coplanar concurrent force system only.

This chapter deals with Resultant force of coplanar collinear and concurrent forces.

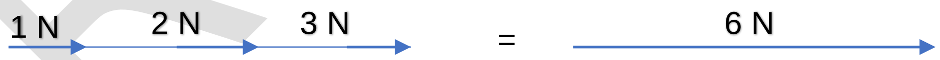
3.2. Resultant force

If the system of forces is replaced by the sinle force having the same effect as that of the original forces. This single force is called Resultant force.

If a number of forces acting on a particle simultaneously are replaced by a single force, which could produce the same effect as produced by the given forces, that single force is called Resultant force. It is an equivalent force of all the given forces.

In the analysis of any structure, a force system can always be replaced by a resultant force so that, the effect of the applied force on the body can easily be studied by studying the effect of their equivalent and single force called resultant

i.e.,



Three collinear forces 1N, 2N and 3N are acting on a line a-b, and in same direction (towards right). These three forces can be replaced by a single force, equals to the sum of the given forces (6N) which acts on the line a-b and in same direction (towards right) as shown in figure. Here, the combined effect of three forces will be equal to the effect of the single force 6N. Hence 6N force is called the Resultant Force of the given forces.

Resultant forces of all the force systems can be determined by two methods

(1) Analytical method. (2) Graphical method

Analytical method

3.3. Resultant force of collinear force

(a) Like collinear forces

Consider three collinear forces P, Q and S acting on a common line of action an in same direction. The magnitude of the resultant force of these three forces is the sum of the forces. Hence, the resultant force is R = P + Q + S.

The line of action of the resultant force is the line of action of given forces. The direction of the resultant force is the same as the direction of the given forces (i.e., upwards). The resultant forces are shown in the figure.

(b) Unlike collinear force

Consider three collinear forces P, Qand S, if the force P and S are acting along the same direction (towards right) and the force Q is in opposite direction (towards left)

Then R = P+ S -Q

For convenience, Upward vertical force, and horizontal force acting from left to right are positive. While downward force and horizontal force acting from right to left are negative.

Resultant force of concurrent forces

Let us find the resultant force of two concurrent forces and resultant force of more than two concurrent forces.

Here, the resultant of two concurrent forces are derived from the parallelogram law of forces as discussed in unit 2

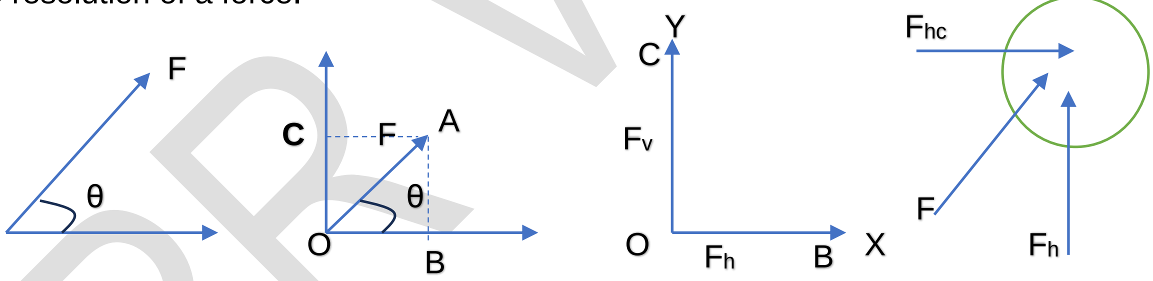
Resultant force of more than two concurrent forces

If more than two concurrent forces acts at a point, we can use the parallelogram law of forces to calculate their resultant. This procedure is repeated until the final resultant is obtained. First any two forces can be taken and their resultant is found by parallelogram law. Then combine the resultant with the third force and again apply parallelogram law. Combine this with the fourth force and get the resultant. This procedure is continued till al the forces have been combined.

Instead, we will apply the principle of Resolution of forces, to find the resultant force of more than two concurrent forces.

3.4. Resolution of a force

A force is either horizontal or vertical. Again a horizontal force may be either right-hand side or left-hand side direction. Similarly, a vertical force may be either upward or downward. Now let us see an inclined force and its sign convention. First of all the given inclined force has to be resolved into two components 1.e., Horizontal and vertical. Splitting up of a force into components along the fixed reference axes is called the resolution of a force.



Consider an inclined force F inclined at angle θ with horizontal as shown in figure. Let us resolve the force into two components, along two fixed axes i.e., OX and OY. Draw the coordinate axes OX and OY through the tail of force O as shown in figure. Through the head of the force A, draw the parallel lines CA and AB parallel to OX and OY axes respectively. Segment OB represents the component of the inclined force F along X axis and OC represents the component of the inclined force along Y axis

Magnitude of components

Let F_h = Horizontal component of the force F

= OB or CA in figure

Fv = Vertical component of the force F

= OC or Ba in figure

Consider the right-angled triangle OAB

$\cos\theta = \frac{OB}{OA} = \frac{F_h}{F} \quad \therefore\ F_h = F\ cos\theta$

Similarly, $\sin\theta = \frac{AB}{OA} = \frac{F_v}{F} \quad \therefore\ F_v = F\sin\theta$

Therefore, if θ is the inclination of a force F with respect to the X axis., then the magnitude of its horizontal and vertical components is F cos θ and F sin θ respectively.

Direction of the components

The resolved component of the force shows that the Horizontal component acts towards the right (Figure C). Hence it takes a positive measure vertical component OC acts upwards. Thus, it takes a positive measure.

As OC and AN are parallel and OB and CA are parallel, the components can also be drawn as shown in figure D. The component F_h acts on (OB) is replaced on CA and the component F_v acts on OC is replaced on BA

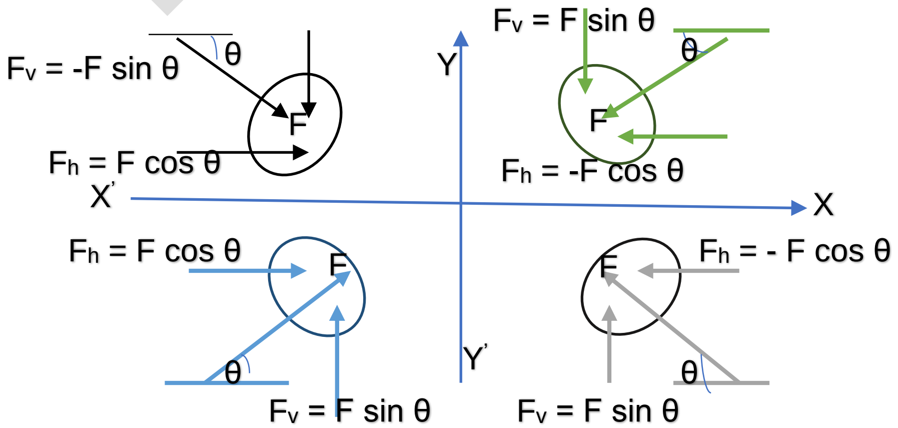
Therefore, to get the sin convention of the components of an inclined force, draw the components, having the arrow heads of components facing the arrow head of the inclined force.

Hence, combining the magnitude and direction, Resolved components of the inclined force shown in figure can be written as follows:

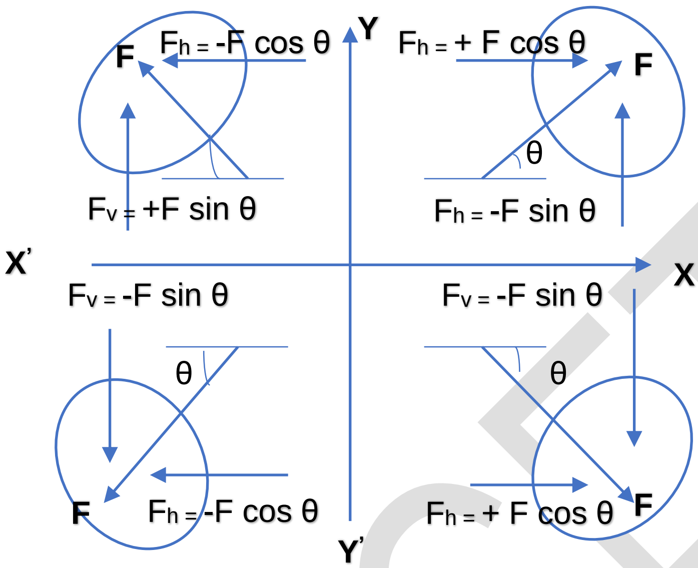
F_h = Horizontal component =+ F cos θ

F_v = Vertical component = + F sin θ

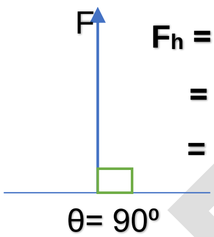
For ready reference, inclined forces, acting towards the point of origin, in all four quadrants are resolved as shown below:



Similarly, the forces acting in all four quadrants, but acting outwards from the point of origin are resolved as shown below:



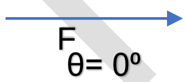
1. A vertical force has no horizontal component; its vertical component is the magnitude of the given force itself.



$$\mathbf{F_h = F \cos\theta = F \cos 90^o = 0}$$

$$\mathbf{F_h = F \sin\theta = F \sin 90^o = F}$$

2. A horizontal force has no vertical component and its horizontal component is the magnitude of the given force itself.



$$\mathbf{F_h = F \cos\theta = F \cos 0 = F}$$

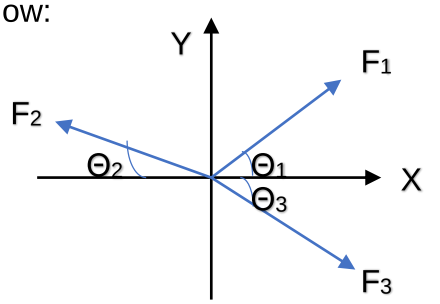
$$\mathbf{F_v = F \sin\theta = F \sin 0 = 0}$$

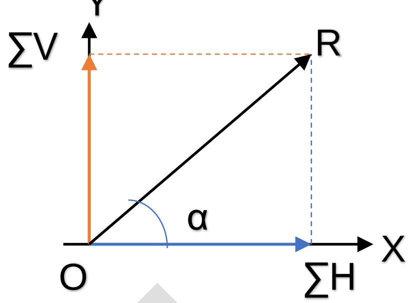
3.5. Procedure to find the resultant force of more than two concurrent force

The step-by-step procedure for finding the resultant force of more than two concurrent forces is given below:

Step 1: Find the algebraic sum of the horizontal components

Resolve the force horizontally and find the net horizontal force, taking right hand side force as positive. Let it be $\sum H$. For example, the concurrent force system as shown below:





Resolving the force horizontally (i.e., along X axis) we get,

$\sum H = F_1 \cos \theta_1 - F_2 \cos \theta_2 + F_3 \cos \theta_3$

Step 2: Find the algebraic sum of the vertical components

Resolving the forces vertically and find the net vertical force, taking upward force as positive. Let it be $\sum V$. Resolving the force vertically (i.e., along Y axis), we get

$\sum V = F_1 \sin \theta_1 + F_2 \sin \theta_2 - F_3 \sin \theta_3$

Step 3: Find the magnitude of the Resultant force

The net horizontal force $\sum H$ (found in step 1) and the net vertical force $\sum V$ (found in step 2) can be drawn in the coordinate axes as shown in the figure.

If $\sum H$ is +ve, draw towards right; $\sum H$ is -ve, draw towards left

If $\sum V$ is +ve, draw upwards; $\sum V$ is -ve, draw downwards.

Assuming both $\sum H$ and $\sum V$ are positive, the magnitude of Resultant force, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

Step 4: Direction of Resultant force

Construct a rectangle with $\sum H$ and $\sum V$ as the adjacent sides. Draw the diagonal originating from the point of origin. This diagonal of the rectangle will be the resultant force of $\sum H$ and $\sum V$ (or) the resultant force with horizontal.

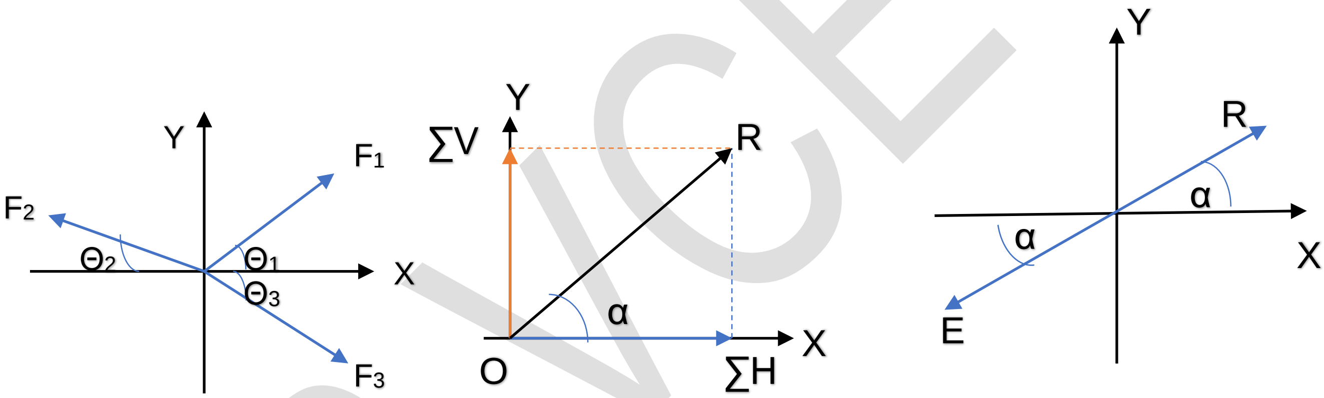
$tan\alpha = \frac{\Sigma V}{\Sigma H}$ (or) $\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

3.6. Equilibrium of particles in two dimensions

3.6.1. Introduction

The effect of the resultant force on the particle is that, the particle starts moving in the direction of the resultant force. But, with the action of force, if the particle does not start moving (or) the particle moves with uniform motion, then the particle is said to be in equilibrium.in other words. *if the resultant of a number of forces acting on a particle is zero, the particle is in equilibrium. The set of forces, where the resultant is zero are called Equilibrium Forces.*

3.6.2 Equilibrium and Equilibrant



Consider a particle subjected to three coplanar concurrent forces as shown in the figure. Let the resultant force of the system is R with direction 'α' with horizontal. Due to this resultant force, the particle may starts moving in the direction of the Resultant force. But if we apply an additional force of some magnitude and direction as that of a resultant force, on the same line of action but in opposite direction, then the movement of the particle will be arrested, or the particle is said to be in equilibrium. The force E, which brings the particle (or set of force) to equilibrium is called an Equilibrant.

Equilibrant (E) is equal to the resultant force (R) in magnitude and direction , collinear but opposite in nature.

3.6.3. Conditions for Equilibrium

For the equilibrium condition of the force system, the resultant is zero. i.e., R = 0

But $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$, Hence both $\sum H$ and $\sum V$ are to be zero for equilibrium condition.

Equation of Equilibrium in Two Dimensions:

$\sum H = 0$ for horizontal collinear forces

$\sum V = 0$ for vertical collinear forces

Both $\sum H = 0$ and $\sum V = 0$ for concurrent forces. These are necessary and sufficient conditions for the equilibrium of particles in two dimensions.

Where $\sum H$ = Algebraic sum of Horizontal forces

and $\sum V$ = Algebraic sum of Vertical forces

$\sum H = 0$ indicates that (← = →) (i.e., sum of left side forces = sum of right side forces)

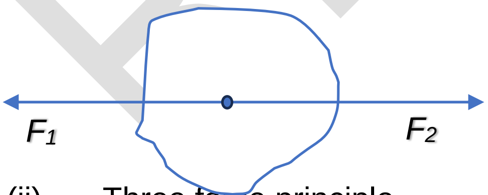
$\sum V = 0$ indicates that (↑ = ↓) (i.e., sum of vertical upward forces = sum of vertical downward forces)

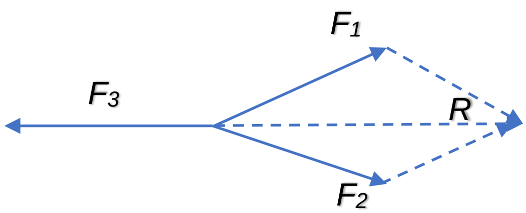
3.6.4. Principle of Equilibrium

Equilibrium principles are developed from the force law of equilibrium (i.e., $\sum F = 0$) From the subject's point of view three principles are important

(i) Two force principle

If a body is subjected to two forces, then the body will be in equilibrium If the two forces are collinear, equal and opposite





(ii) Three force principle

If a body is subjected to three forces, then the body will be in equilibrium, if the resultant of any two forces is equal, opposite and collinear with the third force

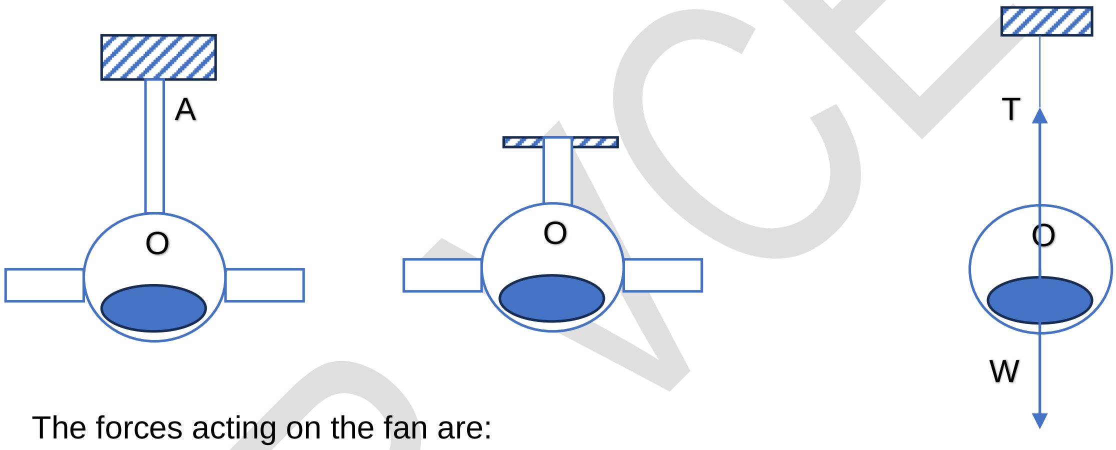
R is the resultant of F_1 and F_2 and also $R = F_3$

(iii) For solving the problem of equilibrium of three coplanar concurrent forces, Lami's theorem is used.

3.7. Free body diagram

In equilibrium analysis of structures / Machines, it is necessary to consider all the forces acting on the body and exclude all the forces which are not directly applied to it. The problem becomes much simpler if each body is considered in isolation i.e., separate from the surrounding bodies is called the Free body. The sketch showing all the forces (both external forces and reactions) and moments acting on the body is called the Free-body diagram.

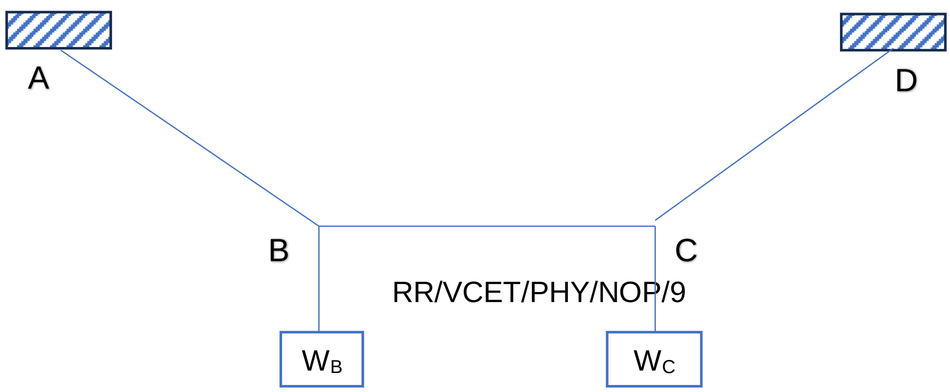
Let us consider a fan of weight 'W' suspended by the string AO from the ceiling as shown in the figure. The weight of the fan is acting through the centre O



The forces acting on the fan are:

1. The self-weight of the fan, which acts vertically downwards. Let it be W
2. To keep the fan in the condition of equilibrium, there must be some upward force in the string AO, which is holding the weight of the fan in its position. Let it be *I.*
3. The free-body diagram of these forces are shown in the last figure. It is to be noted that we have drawn the free-body diagram, of fan only, hence the reaction at A need not be shown. Similarly, we can draw the free body diagram at B and C for the following diagram:

:

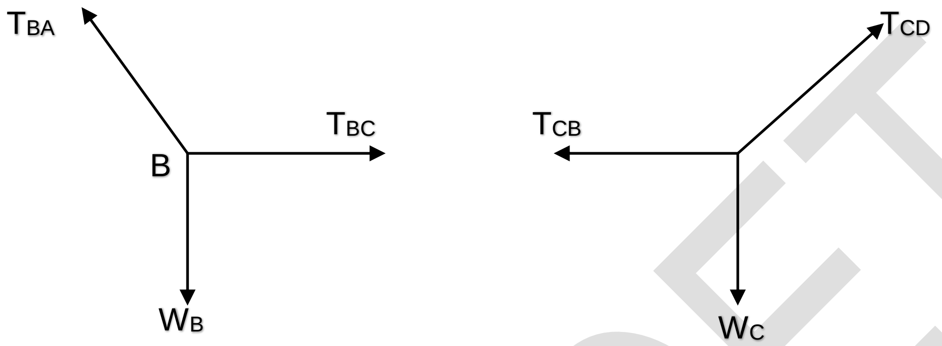


The figure shows two weights, attached at B and C, connected by a string ABCD, supported on the ceiling at A and D

Free body diagram at B

The forces acting at B are:

(i) Weight of the body attached at B, acting downwards, let it be W_B
(ii) Tension on string AB, acting at B and towards A, let it be T_{BA}
(iii) Tension on string BC, acting at B and towards C, let it be T_{BC}

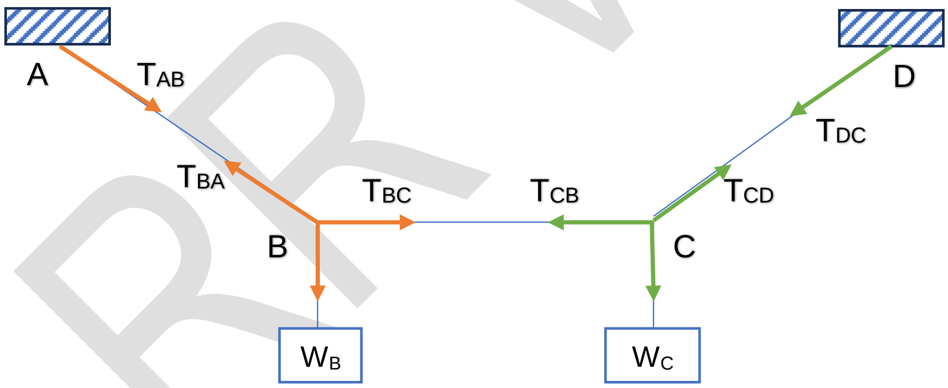


Free body diagram at C

The forces acting at C are:

(i) Weight of the body attached at BC, acting downwards, let it be W_C
(ii) Tension on string CB, acting at C and towards B, let it be T_{CB}
(iii) Tension on string CD, acting at C and towards CD, let it be T_{CD}.

- Now all the forces acting on the string ABCD are shown below:



In string A, there are two forces T_{BA} and T_{AB}, one at each end. For the equilibrium condition of the string, these two forces should be equal , collinear and opposite to each other (as per two force equilibrium principle)

i.e., $T_{BA} = T_{AB}$; $T_{BC} = T_{CB}$ and $T_{CD} = T_{DC}$

To find the unknown forces at B and C, apply the equations of equilibrium (or Lami's theorem) at B and C separately.

In this figure, the reaction at A and D are not shown. To find the unknown forces at B and C, the reactions at A and D are not at all required.

3.8. Action and Reaction

Consider a ball placed on a horizontal surface as shown in the figure. The self-weight of the ball (W) is acting vertically downwards, through its centre of gravity. This force is called "ACTION". Now the ball can move horizontally; but this vertical download motion is resisted due to restoring force developed at support (Here, at the point of contact A) acting vertically upwards. This force is called REACTION. Let it be R_A.

For the condition of equilibrium, Action and Reaction are equal, collinear, but acting in opposite directions (Two force equilibrium)

Table: Free body diagram

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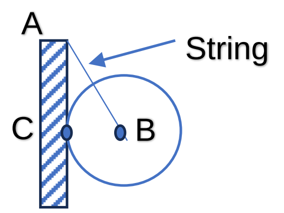
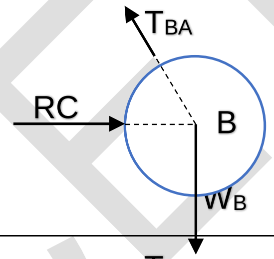
<tr><td>1</td><td>
</td><td>
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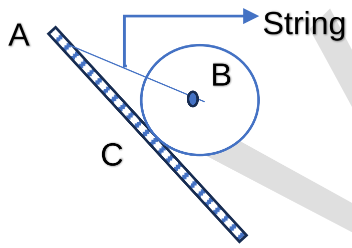
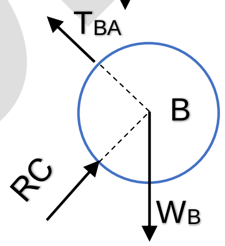
<tr><td>2</td><td>
</td><td>
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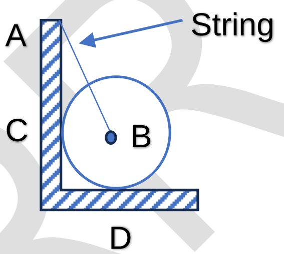
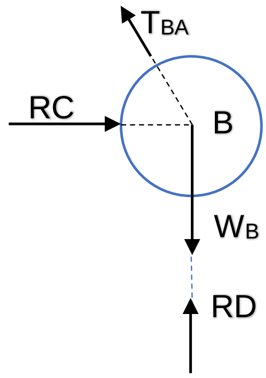
<tr><td>3</td><td>
</td><td>
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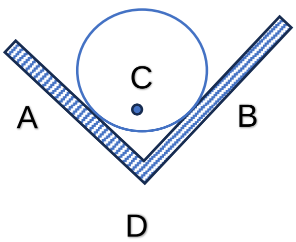
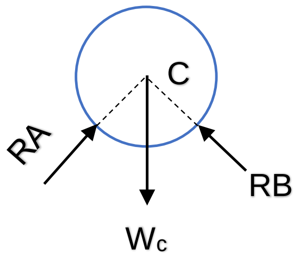
<tr><td>4</td><td>
</td><td>
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T_{BA}(F)

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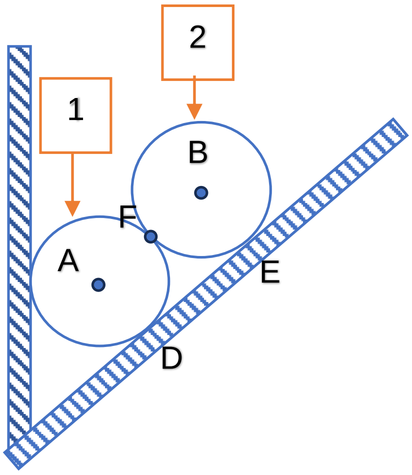
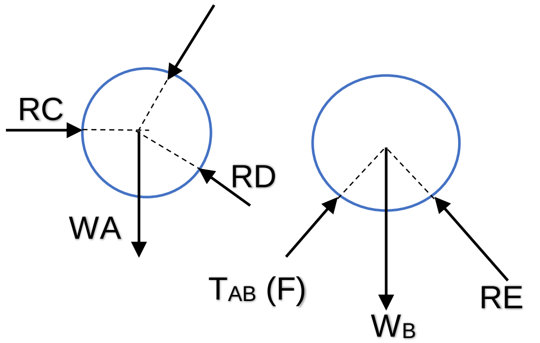
<tr><td>5</td><td>
</td><td>
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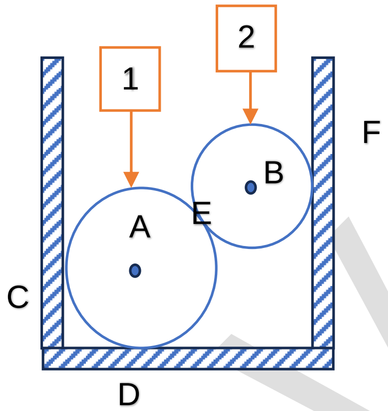
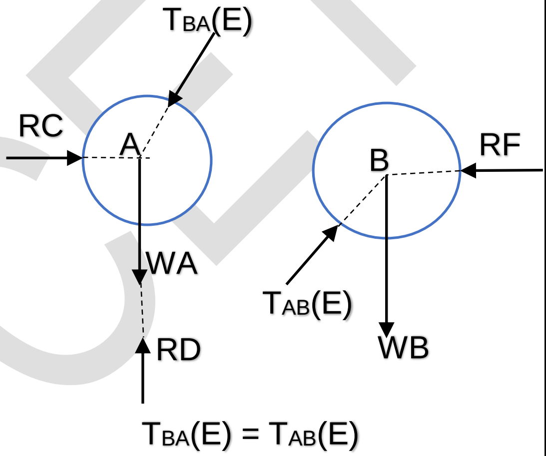
<tr><td>6</td><td>
</td><td>
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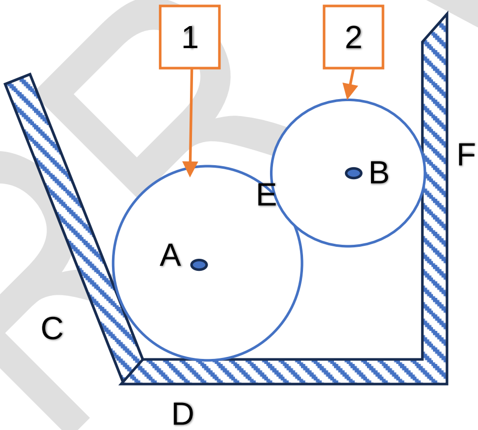
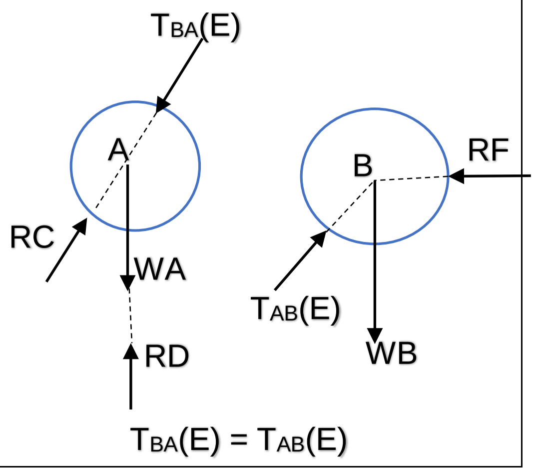
<tr><td>7</td><td>
</td><td>
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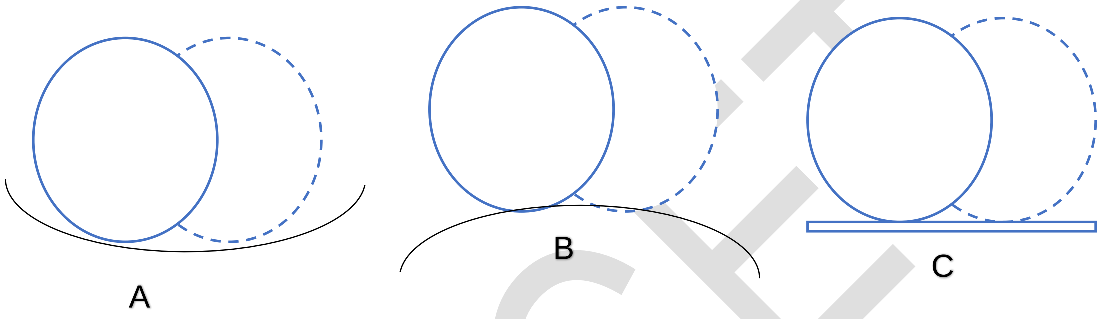
3.9. Types of equilibrium

From the practical point of view, a body is said to be in equilibrium when it returns to its original position after being given a small displacement. This is known as stable equilibrium. Figure A shows the same sphere resting on a convex surface. If a slight displacement disturbs the equilibrium of the sphere, then there is no possibility of it rotating back to its original position by itself. This type of equilibrium is called unstable equilibrium.

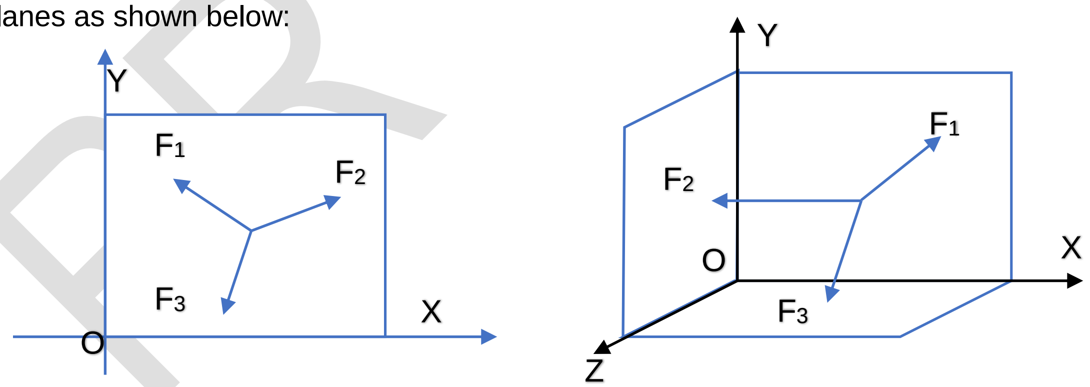
Figure C shows the same sphere resting on a plane's horizontal surface. If a slight displacement is given to the sphere in this position, then it occupies a similar position. In other words, the sphere is neutral, and it occupies a similar position. In other words, the sphere is neutral, and it occupies any position on this horizontal plane depending upon the displacement given to it. This type of equilibrium is called neutral equilibrium.



3.10. Forces in space

3.10.1 Introduction

In the previous chapters we have seen the trigonometrical method of solving the problems of forces in two dimensions. The resultant force and equilibrium of particles in two dimensions is going to discuss here. That is, the external force system acting on a particle lies in one plane and the particle are also subjected to external forces which lies in different planes as shown below:



3.11. Vector approach

The principles of Resultant force and Equilibrium of forces in space are same as that of the forces in plane. But itis very difficult to follow trigonometrical method for the forces in space since finding the scalar components of a force in three dimension is complicated. Hence, vector approach is followed for the force in space. Writing a force in vector notation is more advantageous for solving the problems of forces in space.

To follow vector approach, the knowledge of vector operations is essential. Additional vector operations and important theorems on vector algebra are required in “Vector approach” as discussed in unit 2.