4. Quantum Mechanics

Syllabus

Black body radiation (Qualitative) – Planck's hypothesis - Matter waves – de Broglie hypothesis - Electron microscope – Uncertainty Principle – The Schrodinger Wave equation (time-independent and time-dependent) – Physical significance of wave function - Degenerate energy states - Barrier penetration and quantum tunnelling - Tunneling microscope.

4.1. Introduction

The classical mechanics is not adequate to explain the motion of atomic particles like electrons, protons, etc., many examples of the failure of classical mechanics are:

(1) Black body radiation (ii) specific heat of solids at low temperatures (iii) theory of atomic structure (iv) photoelectric effect and (v) Compton effect.

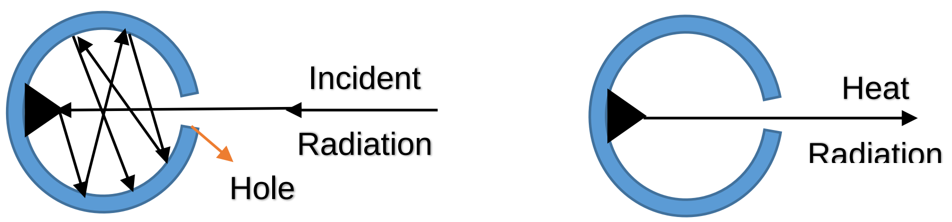
4.2. Black body

In practice, a perfect black body is not available. The body showing a close approximation to an ideal black body can be constructed.

A hollow copper spherical shell is coated with a lamp black on its inner surface. A fine hole is made in this, and a pointed projection is provided just in front of the hole.

When the heat radiation enters into this spherical shell through the hole, the heat radiations suffer multiple reflections and they are completely absorbed. Now the body acts as a observer.

When the body is placed in a constant temperature both at high temperature bath at high temperature, the heat radiations comes out from the hole. Now, this hole acts as a radiator. It is noted that only the hole and not the walls of the body acts as the radiator.



(a) Absorber (b) Radiator

Perfect black body radiation

A perfect black body is one which absorbs all the radiations (all the wavelengths) incident on it. Further, when such a body is placed at constant high temperature, it emits radiation of all the wavelengths.

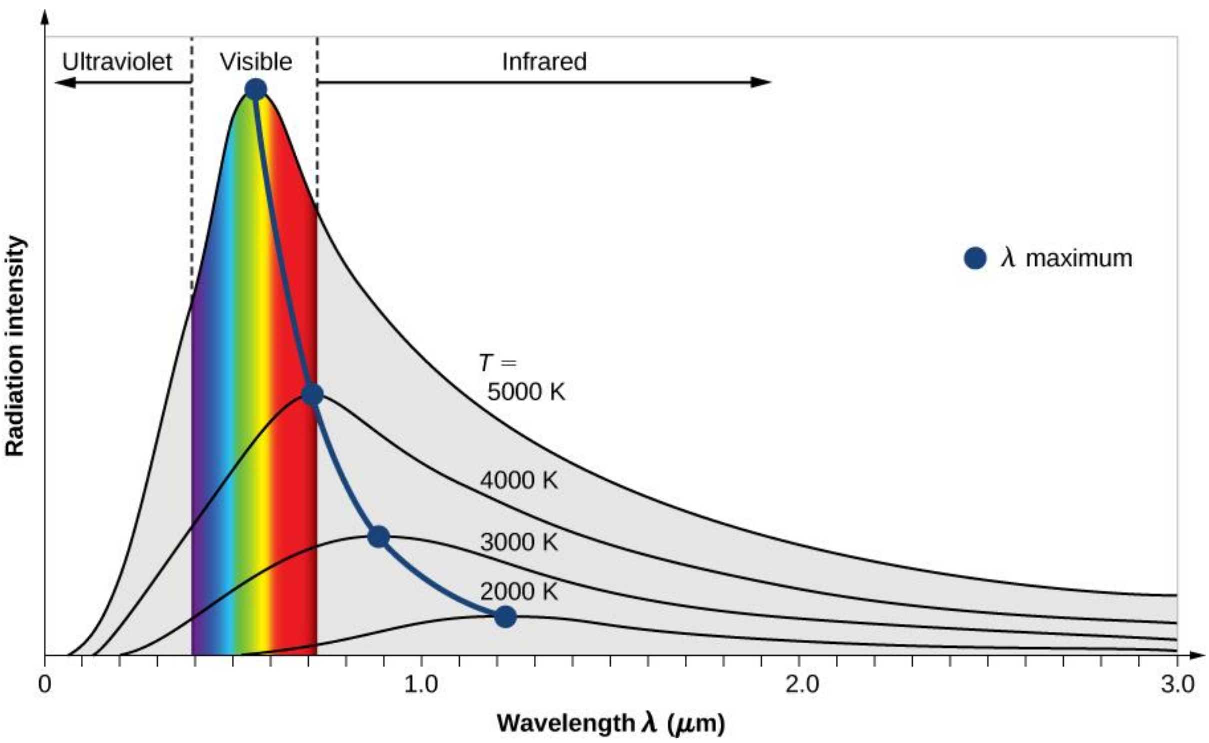
Any object coated with a dull black pigment is a good approximation to a perfect black body.

Black body radiation

The heat radiation emitted from the black body is known as black body radiation. The wavelength at which the maximum energy of radiation emitted depends only on temperature of the black body and it does not depend on the nature of the material.

Energy spectrum of black body radiation

The radiation emitted by a black body varies with its temperature. The intensity of radiation corresponding to different wavelengths is measured at different temperatures and plotted.



The black body spectra is shown in figure. It has the following characteristics:

(i) The black body emits all kinds of radiation ranging from lower wavelength to higher wavelength.

(ii) The black body spectrum shows that the energy density increases with the increases with the increase in wavelength and reaches a maximum value and then decreases with the increase in wavelength.

(iii) The wavelength corresponding to the maximum energy density gets shifted towards lower wavelength, with the increase of temperature.

(iv) If the temperature of the black body is increased, the energy density also increase.

Laws of black body radiation

Wien's displacement law

This law states that the product of wavelength (λ_m) corresponding to the maximum energy of radiation and absolute temperature of the black body (T) is a constant.

i.e., $\lambda_m T = constant$

$$\lambda_m = \frac{constant}{T}$$

$$\lambda_m \, \alpha \, \frac{1}{T}$$

It is found that the wavelength corresponding to the maximum energy of black body radiation is inversely proportional to absolute temperature.

As the temperature of black body increases, the wavelength corresponding to maximum energy decreases.

Wien's radiation law

When deduced the law for the energy emitted by a black body at a given wavelength (λ) and temperature (T) to explain the black body spectrum. It is known as Wien's radiation law.

The energy density in the wavelength range λ and λ + dλ is given by

$$E_\lambda = \frac{8\,\pi\,h\,c}{\lambda^5}\,\frac{1}{e^{\frac{hv}{kT}}} = \frac{8\,\pi\,h\,c}{\lambda^5}\,\frac{1}{e^{\frac{hC}{\lambda kT}}} \qquad \because v = \frac{C}{\lambda}$$

$E_\lambda = 8\,\pi\,h\,c\,\lambda^{-5}\,e^{-hC/\lambda kT}$

$E_\lambda = C_1\lambda^{-5}e^{-C_2/\lambda T}$

Where C_1 and C_2 are constants. T is the temperature of the black body.

The constant C_1 = 8 π h c and C_2 = h C / k

Limitations

This law holds good only for short wavelengths and not for longer wavelengths.

Rayleigh – Jeans law

This law states that the energy distribution of a black body is directly proportional to the absolute temperature (T) and inversely proportional to the fourth power of the wavelength (λ).

i.e., $E_\lambda \ \alpha \ T$

$$E_\lambda \ \alpha \ \frac{1}{\lambda^4}$$

$$E_\lambda \ \alpha \ \frac{T}{\lambda^4}$$

$$E_\lambda = \frac{8\,\pi k\,T}{\lambda^4}$$

Where *k* is a Boltzmann's constant.

Limitation

This law holds good only for longer wavelength regions and not for shorter wavelengths.

It is found that both Wien's and Rayleigh – Jeans laws do not agree with the experimental results for entire wavelength range. Therefore, it is concluded that classical theory failed to explain the emission of black body radiation. Thus, Max-Planck introduced quantum theory to explain laws of the black body radiation.

Planck's law of radiation

Assumptions: Planck derived an expression for the energy distribution, with the following assumptions:

(i) A black body radiator contains electrons or so called simple harmonic oscillators, which are capable of vibrating with all possible frequencies.

(ii) The frequency of radiation emitted by an oscillator is the same as that of the frequency of vibrating particles

(iii) The oscillators (electrons) radiate energy in a discrete manner and not in a continuous manner.

(iv) The oscillators exchanges energy in the form of either absorption or emission within the surroundings in terms of quanta of magnitude '$h\nu$' as in fig. 4.1

(v) The vibrating particles can radiate energy when the oscillator moves from one state to another

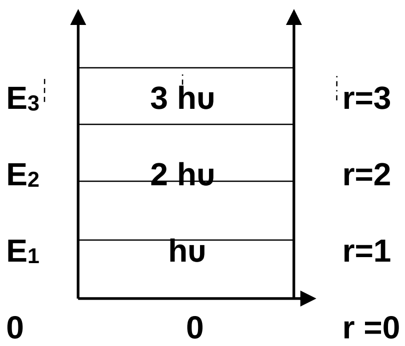


Fig 4.1

i.e., $E = nh\nu$

where n = 0,1,2,3…..

Thus the exchange of energy are limited to a discrete set of values say $0, h\nu, 2h\nu, 3h\nu$ ……. For 0, E, 2E, 3E… for r-oscillators.

DeBroglie hypothesis

According to de-Broglie hypothesis, a moving particle is always associated with waves.

(i) Waves and particles are the only two modes through which energy can propagate in nature

(ii) Our universe is fully composed of light radiation and matter

(iii) Since nature loves symmetry, so matter and waves must be symmetric.

The waves associated with the matter particles are called *matter waves or de-Broglie waves.*

From Planck's theory, the energy of a photon of frequency υ is given by

$$E = h\upsilon \qquad (1)$$

According to Einstein's mass energy relation $E = mc^2$ (2)

Where m – mass of a photon, *c* – velocity of a photon

Equating (1) and (2), we get

$$h\upsilon = mc^2 \qquad (3)$$

$$\frac{hc}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mc} \qquad (4)$$

Since *mc*=*p* momentum of photon, then

$$\lambda = \frac{h}{p} \qquad (5)$$

According to de-Broglie hypothesis, the wavelength of de-Broglie wave associated with any moving particle of mass *'m'* with velocity *'v'* is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad (6)$$

In terms of Energy

We know that K.E (E)= $\frac{1}{2}mv^2$

Multiply *m* by both sides mE= $\frac{1}{2}m^2v^2$

(or) $\sqrt{2mE} = mv$

(or) $\sqrt{2mE} = p$

We know that, $\lambda = \frac{h}{p}$ and hence (or) de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mE}}$

In terms of electrons

We know that kinetic energy in terms of electron volt is given by eV= $\frac{1}{2}mv^2$

Multiply *m* by both sides eV= $\frac{1}{2}m^2v^2$

(or) $\sqrt{2meV} = mv$

(or) $\sqrt{2meV} = p$

We know that, $\lambda = \frac{h}{p}$ and hence

(or) de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2meV}}$

Properties of Matter waves

(i) Matter waves are not electromagnetic waves.

(ii) Matter waves are new kind of waves in which due to the motion of the charged particles, electromagnetic waves are produced.

(iii) Lighter particles will have high wavelength

(iv) Particles moving with less velocity will have high wavelength

(v) The velocity of matter wave is greater than the velocity of light

(vi) Particles moving with less velocity will have high wavelength.

(vii) The velocity of matter wave is not a constant, it depends on the velocity of the particle.

(viii) The velocity of matter wave is greater than the velocity of light

Electron microscope

It is a microscope which uses electron beam to illuminate a specimen and it produces an enlarged image of the specimen. It has very high magnification power and resolving power when compared to optical microscope.

Principle

Like an optical microscope, its purpose is to magnify extremely minute objects. The resolving power of microscope is inversely proportional to the wavelength of the radiation used for illuminating the object under study.

Higher magnification as well as resolving power can be obtained by utilizing waves of shorter wavelength (λ)

Electron microscope uses electron waves whose wavelength is given by the formula $\lambda = \frac{12.25}{\sqrt{V}}$

For V = 10000 V, λ = 0.1225 Å which is extremely short. Electron microscope giving magnification more than 2,00,000 X are common in medical research laboratories.

Types of Electron microscopes:

(i) Scanning Electron Microscope (SEM) (iii) Transmission Electron Microscope (TEM) (iii) Scanning Transmission Electron Microscope (STEM) and (iv) Scanning Tunnelling Microscope (STM)

(i) SCANNING ELECTRON MICROSCOPE (SEM)

Principle

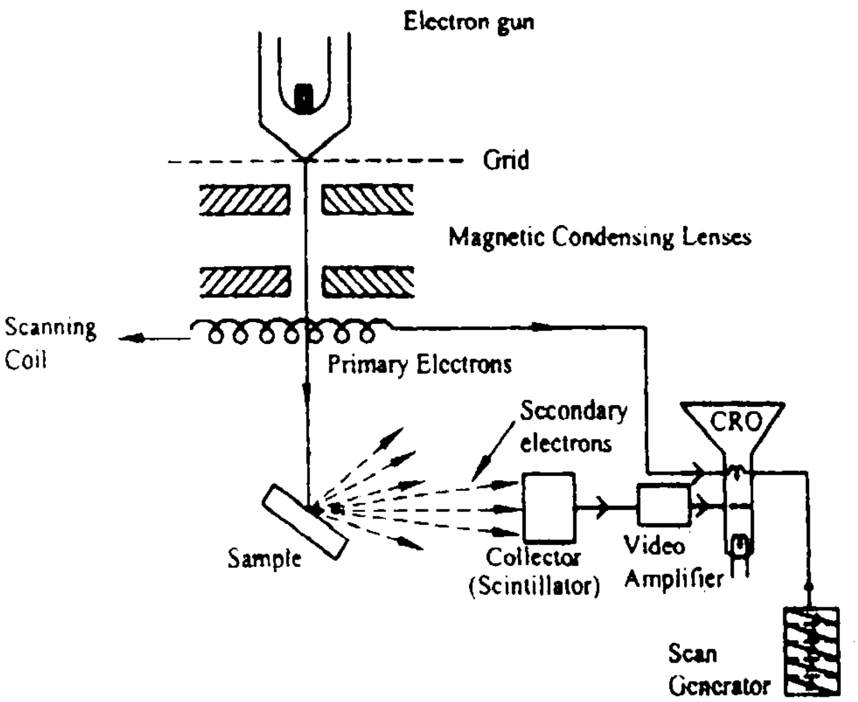
The primary electrons are accelerated and strike the sample, as a result secondary electrons are produced. These electrons are collected by a positive charge electron detector, which gives a three dimensional image of the sample

Construction

It consists of an electron gun to produce high energy electron beam. A magnetic condensing lens is used to condense the electron beam and a scanning coil is arranged in between the magnetic condensing lens and the sample. The electron detector is used to collect the secondary electron and can be converted in to electrical signal. These signals can be fed in to CRO through video amplifier as shown in figure.

Working

Stream of electrons are produced by the electron gun and these primary electrons are accelerated by the grid and anode. These accelerated primary electrons are made to incident on the same through condensing lenses and scanning coil



These high speed primary electrons on falling over the sample produce low energy secondary electrons. The collections of secondary electrons are very difficult because of their low energy. Therefore, to collect these secondary electrons, a very high voltage is applied to the collector

These collected electrons produces scintillation on photo-multiplier (detector) and are converted to electrical signals. These signals are amplified by the video amplifier and is fed to CRO

By similar procedure the electron beam scans the sample from left to right and again from left to right etc., like we read a book and the whole picture of the sample is obtained in the CRO screen

Advantages

(i) It is used to examine a specimen of large thickness

(ii) It has large depth of focus

(iii) It is used to determine the three dimensional image of a object

(iv) Structural details can resolved precisely

(v) Magnification is about 3,00,000 x

Applications

(i) It is used to study the disease causing agent like virus & bacteria

(ii) It is used to study the colloidal particles

(iii) It is used to analyze surface topography of metals

(iv) It is used to determine complicated structure of the crystal

(ii) TRANSMISSION ELECTRON MICROSCOPE

Principle

The electrons are allowed to pass through the specimen and the image is formed on the fluorescent screen either by using transmitted electron beam or by diffracted electron beam from the specimen

Components

(i) Electron gun (ii) Magnetic condensing lens (iii) Fluorescent screen (or) charged couple device

Construction

It consists of an electron gun to produce electrons. Magnetic condensing lens is used to condense the electron beam and it is also used to adjust the size of the electron beam that falls on the specimen

The specimen is kept in between the magnetic condensing lens and magnetic objective lens as shown in figure 4.19. The magnetic objective lens is used to block the high angle diffracted beam and the aperture is used to eliminate the diffracted beam and in turn it increases the contrast of the image

The magnetic projector lens is placed before the fluorescent in order to achieve higher magnification. The image can be recorded by using a fluorescent screen or charged coupled device

Working

The electron beam is produced by the electron gun is made to fall on the specimen using magnetic condensing lens. Based on the angle of incidence, the beam is partly transmitted and partly diffracted, as shown in figure 4.17. Both the transmitted beam and the diffracted beam are combined to form the image. The combined image is called the *phase contrast image*

In order to increase intensity and contrast of the image, an amplitude contrast image has to be obtained. This can be achieved only by using the transmitting beam and thus the diffracted beam has to be eliminated. The resultant beam is passed through the magnetic objective lens and the aperture. The aperture is adjusted in such a way that the diffracted image is eliminated

Thus, the final image obtained due to the transmitted beam alone is passed through the projector lens for further magnification. The magnified image is recorded in the fluorescent screen (or) CCD. This high contrast image is called Bright field image. This image is only due to transmitted beam alone

Advantages

(i) The resolution and magnification is as high as optical microscopes

(ii) The focal length of the lens is altered by changing the current in electromagnet

Limitations

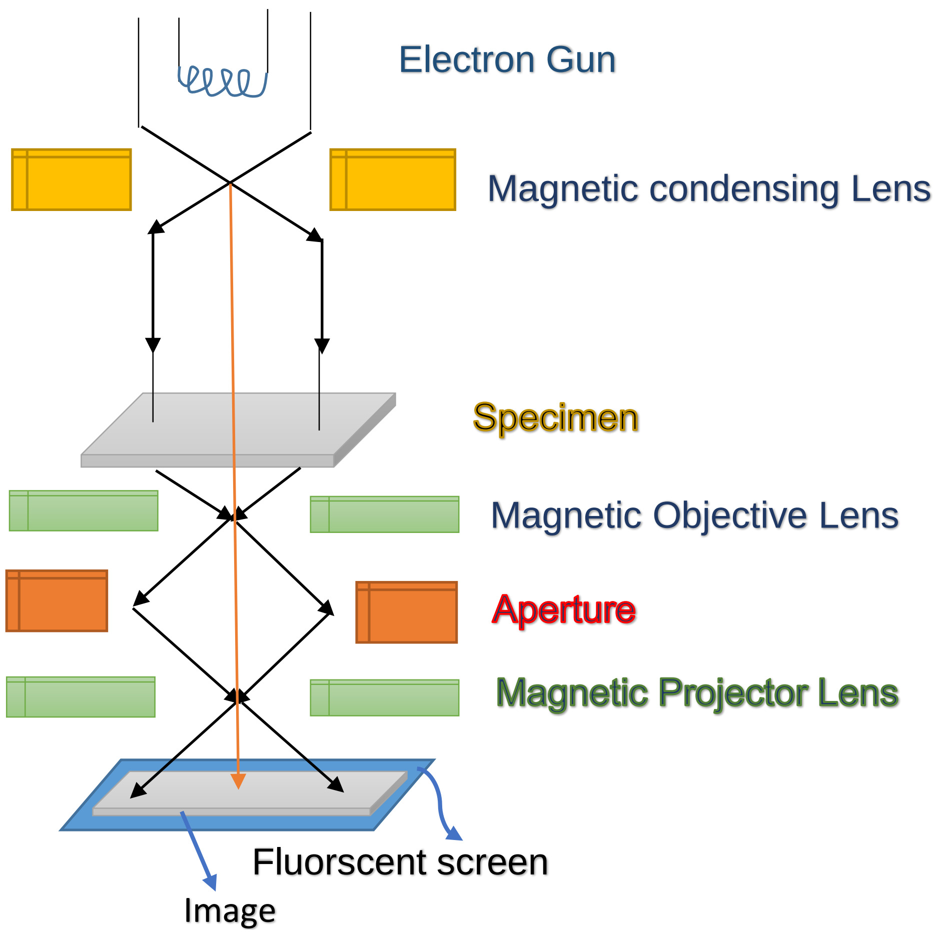
(i) The material require extensive preparation such that the sample is thin enough which is electron transparent

(ii) It is time consuming process

(iii) The structure may change during preparation

(iv) The analyzing area of the sample should be small

(v) Sample may get damaged due to electron beam



Applications

(i) It is used in the investigation of atomic structure and structure of the crystal.

(ii) In biological applications, it is used to create tomographic reconstructions of small cells or thin sections of larger cells

(iii) In material science, it is used to determine the dimensions of powders or nanotubes

(iv) It is used to locate the position of the defects and also the nature of the defect present

Schrödinger time dependent wave function

A particle can behave as a wave only under motion. So, it must be accelerated by a potential field

∴, Total energy (E) = Potential Energy (V) + Kinetic Energy

i.e., $E = V + \frac{1}{2}mv^2$

(or) $E = V + \frac{1}{2}\frac{m^2v^2}{m}$

(or) $E = V + \frac{p^2}{2m}$ [Since p = mv]

(or) $E\Psi = V\Psi + \frac{p^2}{2m}\Psi$ (1)

According to classical mechanics if 'x' is the position of the particle moving with the velocity 'v', then the displacement of the particle at any time 't' is given by

$$y = Ae^{-i\omega\left(t-\left(\frac{x}{v}\right)\right)}$$

Where ω is the angular frequency of the particle

Similarly in quantum mechanics the wave equation Ψ (x, y, z, t) represents the position (x, y, z) of a moving particle at any time't' and is given by

$$\Psi(x,y,z,t) = Ae^{-i\omega\left(t-\left(\frac{x}{v}\right)\right)} \quad (2)$$

We know that angular frequency ω = 2πν

∴ Equation (2) becomes

$$\Psi(x,y,z,t) = Ae^{-i2\pi\left(\upsilon t-\left(\frac{x\upsilon}{v}\right)\right)} \quad (3)$$

We know $E = h\upsilon$ (or) $\upsilon = \frac{E}{h}$ (4)

Also, if 'v' is the velocity of the particle behaving as a wave,

Then the frequency $\upsilon = \frac{v}{\lambda}$ (or) $\frac{v}{\upsilon} = \frac{1}{\lambda}$ (5)

Substituting equations (4) & (5) in equation (3), we get

$$\Psi(x,y,z,t) = Ae^{-i2\pi\left(\left(\frac{Et}{h}\right)-\left(\frac{x}{\lambda}\right)\right)} \quad (6)$$

If 'p' is the momentum of the particle, then the de-Broglie wavelength

is given by $\lambda = \frac{h}{mv} = \frac{h}{p}$ (7)

Substituting equation (7) in (6) we get

$$\Psi(x,y,z,t) = Ae^{-i2\pi\left(\left(\frac{Et}{h}\right)-\left(\frac{px}{h}\right)\right)}$$

(or) $\Psi(x,y,z,t) = Ae^{-i\frac{2\pi}{h}(Et-px)}$

Since $\hbar = \frac{h}{2\pi}$ we can write $\Psi(x,y,z,t) = Ae^{-\frac{i}{\hbar}(Et-px)}$ (8)

Differentiating equation (8) partially with respect to 'x' we get

$$\frac{\partial \Psi}{\partial x} = Ae^{-\frac{i}{\hbar}(Et-Px)}\left(\frac{ip}{\hbar}\right)$$

Differentiating once again partially with respect to 'x' we get

$$\frac{\partial^2 \Psi}{\partial x^2} = Ae^{-\frac{i}{\hbar}(Et-Px)}\left(\frac{i^2 p^2}{\hbar^2}\right)$$

Since $\Psi(x,y,z,t) = Ae^{-\frac{i}{\hbar}(Et-px)}$ and $i^2 = -1$, we can write

$$\frac{\partial^2\Psi}{\partial x^2} = \Psi(x,y,z,t)\left(\frac{-p^2}{\hbar^2}\right)$$

(or) $p^2\Psi = -\hbar^2 \frac{\partial^2\Psi}{\partial x^2}$ (9)

Differentiating equation (8) partially with respect to'‘t’ we get

$$\frac{\partial\Psi}{\partial t} = Ae^{-\frac{i}{\hbar}(Et-Px)}\left(\frac{-iE}{\hbar}\right)$$

(or) $\frac{\hbar}{-i}\frac{\partial\Psi}{\partial t} = \Psi(x,y,z,t)E$ $\left[\because \Psi(x,y,z,t) = Ae^{-\frac{i}{\hbar}(Et-px)}\right]$

(or) $E\Psi = i\hbar\frac{\partial\Psi}{\partial t}$ (10)

Substituting equations (9) & (10) in equation (1) , we get

$$i\hbar\frac{\partial\Psi}{\partial t} = V\Psi - \frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}$$

(or) $i\hbar\frac{\partial\Psi}{\partial t} = \left[V - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right]\Psi$ (11)

Equation (11) represents the one dimensional Schrodinger time dependent wave equation along ‘x’ direction. Also the wave function Ψ (x, y, z, t) depends on both the position (x, y, z) and time (t)

Similarly for three dimensional Schrodinger time dependent wave equation can be written as

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[V - \frac{\hbar^2}{2m}\nabla^2\right]\Psi \qquad (12)$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Equation (12) can also rewritten as EΨ = HΨ

Where E is an energy operator given by $E = i\hbar \frac{\partial}{\partial t}$ &

H is called Hamiltonian operator, given by $H = V - \frac{\hbar^2}{2m}\nabla^2$

Schrödinger time independent wave equation

In Schrödinger time dependent wave equation the wave function 'Ψ' depends on time, but in Schrodinger time independent wave function 'Ψ' does not depend on time & hence it has many applications

We know that time dependent wave function

$$\Psi(x, y, z, t) = Ae^{-\frac{i}{\hbar}(Et - px)}$$

Now, splitting the RHS of this equation in to (i) Time dependent factor & (ii) Time independent factor, we get

$$\Psi(x, y, z, t) = Ae^{\frac{-iEt}{\hbar}} e^{\frac{ipx}{\hbar}}$$

(or) $\Psi(x, y, z, t) = A\,\psi e^{-\frac{iEt}{\hbar}}$. $\Psi(x, y, z, t) = A\psi e^{\frac{-iEt}{\hbar}}$ (1)

Where 'ψ' represents the time independent wave function. i.e., $\psi = e^{\frac{ipx}{\hbar}}$

Differentiating equation (1) partially with respect to 't' we get $\frac{\partial \Psi}{\partial t} = A\psi e^{\frac{-iEt}{\hbar}}\left[\frac{-iE}{\hbar}\right]$ (2)

Differentiating equation (1) partially with respect to 'x' we get,

$$\frac{\partial \Psi}{\partial x} = Ae^{\frac{-iEt}{\hbar}} \frac{\partial \psi}{\partial x}$$

Differentiating once again partially with respect to 'x' we get, $\frac{\partial^2 \Psi}{\partial x^2} = Ae^{\frac{-iEt}{\hbar}} \frac{\partial^2 \psi}{\partial x^2}$ (3)

We know the time dependent wave equation for 1-dimension is

$$i\hbar \frac{\partial \Psi}{\partial t} = V\Psi - \frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} \quad (4)$$

We can get the Schrödinger time dependent wave equation, just by substituting equations (1),(2) & (3), which has relation between the time dependent wave function (Ψ) and time independent wave

Function (ψ) in equation (4)

Thus, substituting equations (1),(2) & (3) in equation (4) , we get

$$i\hbar A\psi e^{\frac{-iEt}{\hbar}} \left[\frac{-iE}{\hbar}\right] = VA\psi e^{\frac{-iEt}{\hbar}} - \frac{\hbar^2}{2m} Ae^{\frac{-iEt}{\hbar}} \frac{\partial^2 \psi}{\partial x^2}$$

(or) $i\hbar\psi \left[\frac{-iE}{\hbar}\right] = V\psi - \frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}$

(or) $(-i)^2 E\psi = V\psi - \frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}$ (or) $E\psi - V\psi = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}$

(or) $\frac{\partial^2 \psi}{\partial x^2} = \frac{-2m}{\hbar^2} [E\psi - V\psi]$

(or) $\frac{\partial^2\psi}{\partial x^2}+\frac{2m}{\hbar^2}\left[E-V\right]\psi=0$ (5)

Equation (5) represents the Schrodinger time independent wave function in one dimension along ‘x’ direction. Here the wave function is independent of time .Similarly for 3 – dimension, the Schrodinger time independent wave function is given by

$\nabla^2\psi+\frac{2m}{\hbar^2}\left[E-V\right]\psi=0$ (6)

Where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$

Physical Significance of a wave function [Ψ]

(i) It gives the relation between the particle and wave nature of the matter statistically

i.e., Ψ = ψ $e^{-i\omega t}$

(ii) Wave function gives the information about the particle behavior

(iii) Ψ is a complex quantity and does not have any physical meaning

(iv) $|\psi|^2=\dot{\psi}^*\psi$ is real & positive. This concept is similar to light. In light amplitude may be (+ve) or (-ve) but the square of intensity of light is +ve & measurable

(v) $|\psi|^2$ represents the probability density of finding the particle per unit volume

(vi) for a given volume $d\tau$, the probability of finding the particle is given by Probability (P) = $\iiint\ |\psi|^2\,d\tau$ where $d\tau=dx.dy.dz$

(vii) The probability will have any values between 0 & 1

1) If P =0 , then there is no particle within the given limits
2) If P = 1 ,the particle is definitely present within the given limits
3) If P = 0.7, then there is 70% chance of finding the particle within the given limits. Also there is 30% of no chance of finding the particle

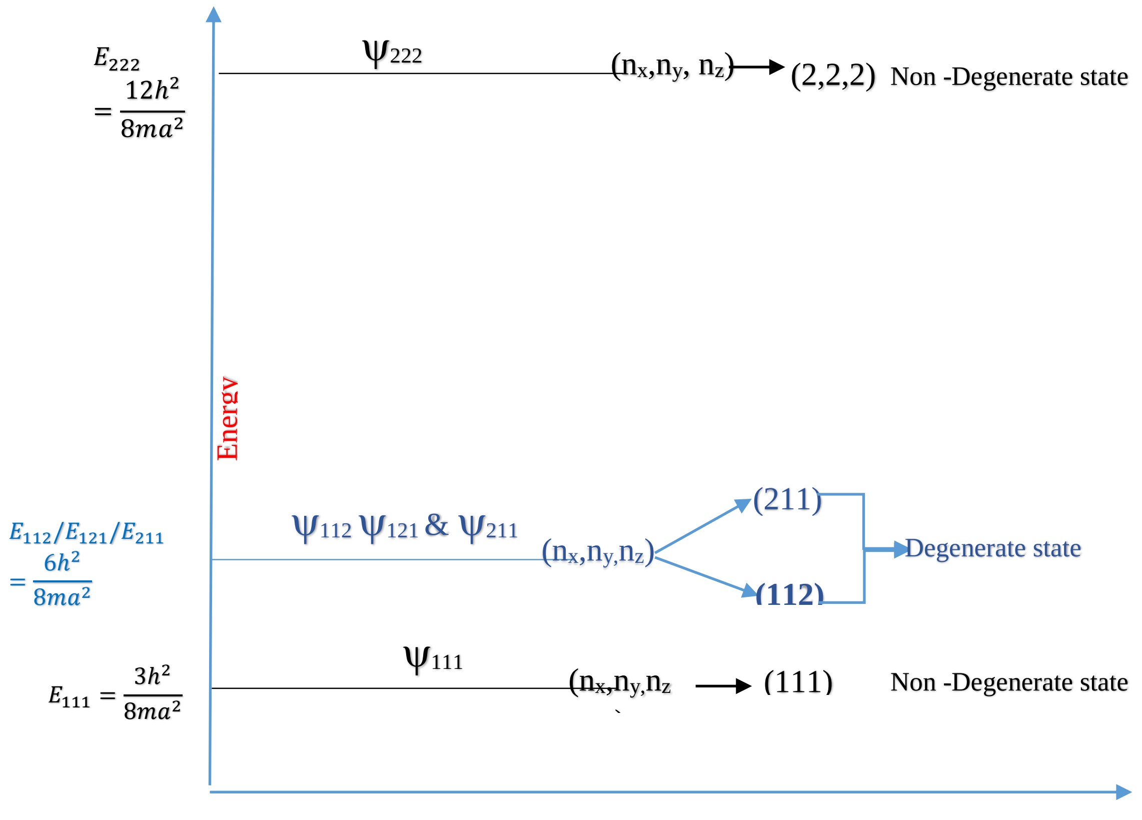
Degeneracy and Non – Degeneracy

Degeneracy

It is seen from equations (30) & (31) for several combination of quantum numbers we have same energy eigen values but different eigen functions. Such states and energy levels are called Degenerate states. The three combination of quantum numbers (112), (121) and (211) which gives same eigen value but different eigen functions are called *3 – fold degenerate state.*

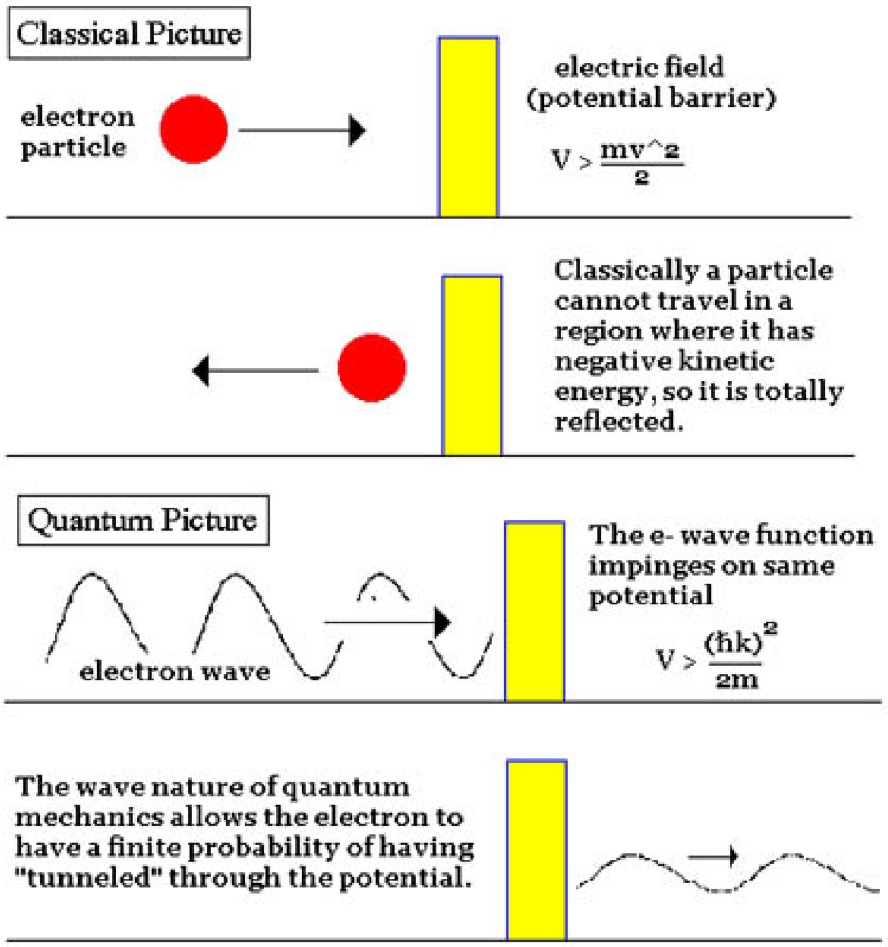
Non – degeneracy

For various combinations of quantum number if we have same energy eigen value and same eigen function (one) then such states and energy levels are called *non – degenerate state.*



Barrier penetration and quantum tunneling (Qualitative)

- According to classical ideas, a particle striking a hard wall has no chance of leaking through it. But, the behaviour of a quantum particle is different due to the wave nature associated with it.
- We know that when an electromagnetic wave strikes at the interface of two media, it is partly reflected and partly transmitted through the interface and enters the second medium.
- In a similar way the de Broglie wave also has a possibility of getting partly reflected from the boundary of the potential well and partly penetrating through the barrier.
- Figure shows a particle with energy $E<V$ approaching potential barrier of height V.
- An electron of total energy E approaches the barrier from the left. From the view point of classical physics, the electron would be reflected from the barrier because its energy E is less than V.
- For the particle to overcome the potential barrier, it must have an energy equal to or greater than V.



Quantum mechanics leads to an entirely new result. It shows that there is a finite chance for the electron to leak to the other side of the barrier. It is noted that the electron tunnelled through the potential barrier and hence in quantum mechanics, this phenomenon is called tunneling. The transmission of electrons through the barrier is known as barrier penetration.

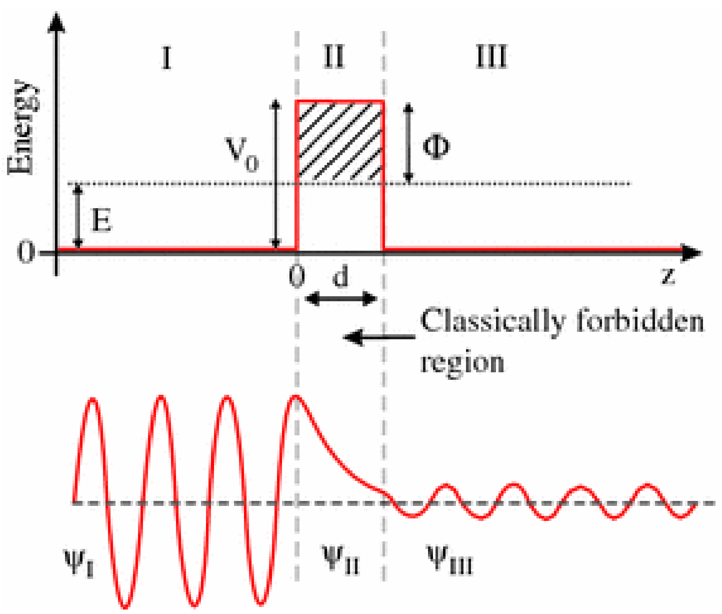
Expression for Transmission Probability

- Now let us consider the case of a particle of energy E<V approaching a potential barrier of finite height and width as shown in figure.
- The particle in region *I* has certain probability of passing through the barrier to reach region II and then emerge out on the other side in region III.
- The particle lacks the energy to go over the top of the barrier, but tunnels through it. Higher the barrier and wider it is, the lesser is the probability of the particle tunneling through it.

- Let us now consider a beam of identical particles, all having kinetic energy E. The beam is incident on the potential barrier of height V and width *a* from height I.
- On both sides of the barrier V= 0, This means that no forces act on particles in regions I and III.
- Here, the wave function ψ_1 represents the particle moving towards the barrier from region I while ψ_1 represents the particle reflected moving away from the barrier.
- The wave function ψ_{II} represents the particle inside the barrier. Some of the particles end up in the region III while the other return to region I.
- Quantum mechanics shows that the transmission probability T for a particle to pass through the barrier is given by $T = \frac{\text{Number of particles transmitted}}{\text{Number of particles incident}}$

This is approximately given by $T = T_0 e^{-2ka}$

Where $k = \frac{\sqrt{2m(V-E)}}{h}$ and *a* is the width of the barrier. T_0 is a constant close to unity. It shows that the probability of particle penetration through a potential barrier depends on the height and width of the barrier.



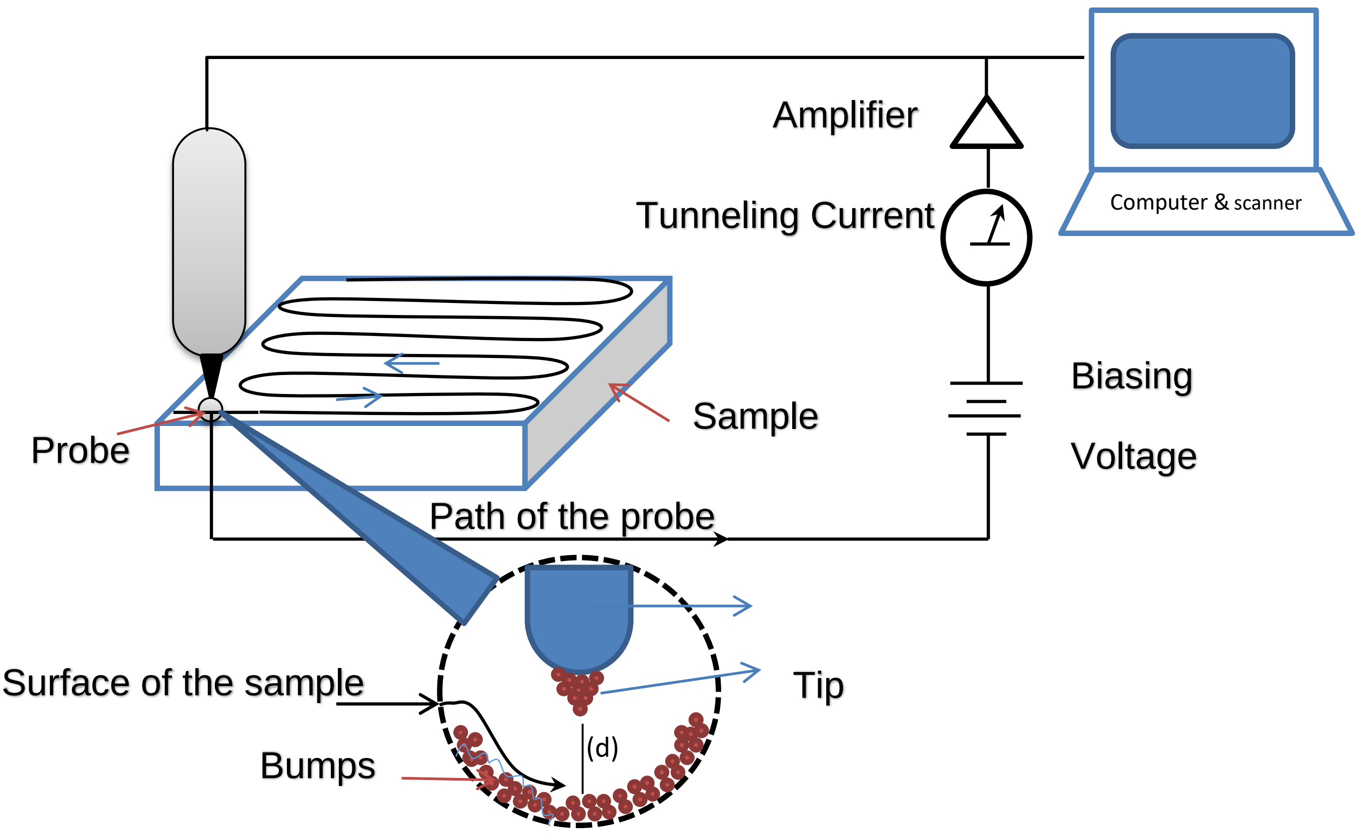
Significance

Tunneling is a very important physical phenomena which occurs in certain semiconductor diodes. In such diodes electrons pass through potential barriers even though their kinetic energies are smaller than the barrier heights.

The tunneling effect also occurs in the case of the alpha particles. The kinetic energy of alpha particle is only a few MeV but it is able to escape from a nucleus whose potential wall is perhaps 25 MeV high.

The ability of electrons to tunnel through a potential barrier is used in the Scanning Tunneling Microscope (STM) to study surfaces on an atomic scale of size.

Scanning Tunneling Microscope (STM)



Principle

The tunneling of electron between the sharp metallic tip of the probe and the surface of the sample. Here the tunneling current is maintained by adjusting the distance between the tip and the sample, with an air gap for electron to tunnel. IN a similar way the tip is used to scan atom by atom and line by line of the sample and the topography of the sample is recorded in the computer

Construction

(i) The experimental setup consists of a probe in which a small thin metal wire is etched in such a way that the tip of the probe will have only one atom as shown in figure.

(ii) The tip is tapered down to a single atom, so that it can follow even a small change in the contours of the sample.

(iii) The tip is connected to the scanner and it can be positioned to X, Y, Z coordinates using a personal computer, as shown in figure.

(iv) The sample for which the image has to be recorded is kept below the tip of the probe at a particular distance (atleast to a width of 2 atoms spacing) in such a way that the tip should not touch the sample. i.e., A small air gap should always be maintained between the tip of the probe and the sample.

(v) The computer is also used to record the path of the probe and the topography of the sample in a grey-scale (or) colour. Necessary circuit connections along with an amplifier are provided to measure the tunneling current in the circuit

Working

1. Circuit is switched ON and necessary biasing voltage is given to the probe.
2. Due to biasing the electrons will tunnel (or) jump between the tip of the probe and the sample and therefore produces a small electric current called tunneling current as show in figure.

3. The tunneling current flows through the circuit only if the tip is in contact with the sample through the small air gap at a distance '*d*' between them.
4. The current produced is amplified and measured in the computer
5. It is found that the current increases (or) decreases based on the distance between the tip of the probe and the sample
6. The current in the circuit should be monitored in such a way that it should be maintained constant
7. Therefore, for maintaining the constant current, the distance (*d*) between the tip and the sample should be continuously adjusted, whenever the tip moves over the surface of the sample.
8. The height fluctuations (*d*) between the tip and the sample is accurately recorded and as a resultant, a map of '*bumps*' is obtained in the computer as shown in figure.
9. In a similar way the tip is scanned atom by atom and line by line of the sample and the topography of the sample is recorded in the computer.
10. The STM does not show the picture of the atom, rather it records only the exact only the exact position of the atoms, more precisely the position of electrons.

Advantages

1. It can scan the positions & topography atom by atom (or) even electrons
2. It is the latest technique used in research laborites for scanning the materials
3. very accurate magnification up to nan-scale shall be measured.

Disadvantages

1. Even a very small vibrations will deviate the measurement setup
2. it should kept in vacuum, because even a single dust particle will damage the tip of the probe
3. cost is high

Applications

1. it is used to produce integrated circuit
2. it is used in biomedical devices
3. they are used in materials science studies for both bump and flat surfaces