2. Carriers and Doping

Syllabus

Intrinsic concentration - intrinsic Fermi level - n and p type doping - density of carriers in extrinsic semiconductors and their temperature dependence - extrinsic semiconductor Fermi energy level - degenerate and non - degenerate semiconductors - Direct and Indirect band gap semiconductors - Hall Effect - Schottky junction - Ohmic contacts - Schottky diode.

2.1. Introduction

Semiconducting material has electrical conductivity between a good conductor and a good insulator. It is a special class of material which is very small in size and sensitive to heat, light and electricity. Semiconducting materials behave as insulator at low temperature an as conductor at high temperature. Moreover, these materials have two types of charge carriers i.e., electrons and holes.

Germanium and Silicon are the two elemental semiconductors used in diodes an transistors.

Gallium arsenide (GaAs) and Indium phosphide (InP) are the two compound semiconductors used in LEDs and Laser diodes.

The study of semiconducting materials opened a new branch of technology called solid state electronics due to their wide applications in semiconductor devices in engineering & technology. It leads to the development of IC’s, microprocessors, computers, etc.,

2.2. Definition (based on electrical resistance)

Semiconductor has electrical resistance which is lesser than an insulator but more than that of conductor. Its electrical resistivity is in the order of 10^{-4} to 0.5 ohm meter.

Based on Energy band

A semiconductor has nearly an empty conduction band and almost filled valence band with a very small energy bandgap (~1eV)

Properties

- They have crystalline structure.
- Bonding between the atoms is formed by covalent band
- They have empty conduction band at 0 K
- They have almost filled valence band
- The conductivity of the semiconductor increases due to the temperature & impurity.
- They have negative temperature coefficient of resistance.
- In semiconductors both the electron and holes are charge carriers and will take part in conduction

This property is in contrary to that of metals in which if temperature / impurity increases, their electrical resistivity decreases.

2.3. Classification of semiconductors

Semiconductors are of two types and are classified on the basis of the composition of materials:

Elemental semiconductors, and

Compound semiconductors

1. Elemental semiconductors

The semiconductors which are made from single element of fourth group elements in periodic table are known as elemental semiconductors. They are also called as indirect bandgap semiconductors. Example: Silicon (E_g = 072 eV), Germanium (E_g = 1.1 eV)

2. Compound Semiconductors

The semiconductors which are combined from third and fifth group or second and sixth group elements in periodic table are known as compound semiconductors. They are also called as direct bandgap semiconductors.

Characteristics

It has large forbidden bandgap and mobility

They are formed by ionic and covalent bonds.

Recombination of electron and hole takes place directly.

Uses

It is used in photovoltaic cell, photoconductive cell, LEDs and Laser diodes.

Differences between Elemental and compound semiconductors.

S.No	Elemental Semiconductor	Compound Semiconductor
1.	Made of single element (Si, Ge)	Made of compounds (GaAs, InP)
2.	Indirect band gap semiconductor	Direct bandgap semiconductors
2.	Heat produced during recombination	Photons emitted during recombination
3.	Life time of charge carriers is more	Life time of charge carriers is less
4.	They are used for making diodes, transistors, etc.,	They are used for making LED's, Laser diodes and IC's, etc.,
5.	Current amplification is more	Current amplification is less

2.4. Types of semiconductors

Semiconductors are of two types and are classified on the basis of the concentration of electrons and holes in the materials:

- Pure or intrinsic semiconductors, and
- Doped or extrinsic semiconductors

2.5. Pure or Intrinsic semiconductors

Highly pure semiconductors are called intrinsic semiconductors, which means that the concentration of electrons must be equal to the concentration of holes.

The number of charge carriers per unit volume of the material is called carrier concentration or density of charge carriers

Electrons in Conduction Band

The number of electrons whose energy lies in the range "E" & "E+dE" in the conduction band is given by

$$dn = Z(E)\,F(E)\,dE \qquad (1)$$

Where Z (E) – density of states in the energy ranges 'E' &'E + dE'
F (E) – Probability of number of electron occupying in the conduction band

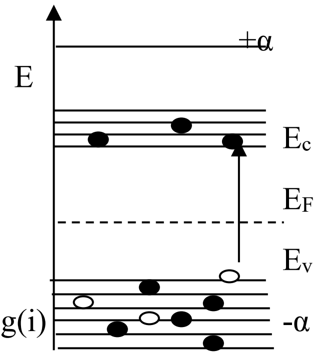
The number of electrons in the conduction band for the entire region is calculated by integrating equation (1) from top energy level 'E_c' to bottom energy level 'α'

i.e., $$n = \int_{E_c}^{\infty} Z(E)F(E)dE \qquad (2)$$

W.K.T, Density of states in the conduction band in the energy range 'E' & 'E+dE' is given by

$$Z(E)\,dE = \frac{4\pi}{h^3}(2m)^{\frac{3}{2}} E^{\frac{1}{2}} dE \qquad (3)$$

The bottom edge of the conduction band (E_c) represents the potential energy of an electron at rest. Therefore, (E - E_c) is the kinetic energy of the conduction electron at higher energy levels. Therefore the equation (3) is modified as



Fig(i)

$$Z(E)\,dE = \frac{4\pi}{h^3}\left(2m_e^*\right)^{\frac{3}{2}}\left(E - E_c\right)^{\frac{1}{2}} dE \qquad (4)$$

Since the electron is moving in a periodic potential, its mass m is replaced by its effective mass $m_e{}^*$.

The probability of electron occupancy is given by $F(E) = \frac{1}{1+e^{(E-E_F)/kT}}$ (5)

Substituting (4) & (5) in (2), we get, $n = \int_{E_c}^{\infty} \frac{4\pi}{h^3}(2m_e^*)^{\frac{3}{2}}(E-E_c)^{\frac{1}{2}} \times \frac{1}{1+e^{(E-E_F)/kT}}\, dE$ (6)

For all possible temperature, $E - E_F >> kT$, hence in the denominator, $1 + e^{\frac{E-E_F}{kT}} \cong e^{\frac{E-E_F}{kT}}$

Now equation (6) becomes, $n = \frac{4\pi}{h^3}(2m_e^*)^{\frac{3}{2}} \int_{E_c}^{\infty} \frac{(E-E_c)^{\frac{1}{2}}}{e^{(E-E_F)/kT}}\, dE$

(or) $$n = \frac{4\pi}{h^3}(2m_e^*)^{\frac{3}{2}} \int_{E_c}^{\infty} (E-E_c)^{\frac{1}{2}} e^{(E_F-E)/kT}\, dE$$

(or) $$n = \frac{4\pi}{h^3}(2m_e^*)^{\frac{3}{2}} e^{E_F/kT} \int_{E_c}^{\infty} (E-E_c)^{\frac{1}{2}} e^{-E/kT}\, dE \qquad (7)$$

To solve this, assume the following:

$E - E_c = x$	$E = E_c$	$E = +\alpha$
$E = E_c + x$	$E_c - E_c = x$	$\alpha - E_c = x$
$dE = dx$	$x = 0$	$x = \alpha$

Now, $$n = \frac{4\pi}{h^3}(2m_e^*)^{\frac{3}{2}} e^{E_F/kT} \int_{0}^{\infty} x^{\frac{1}{2}} e^{-(E_c+x)/kT}\, dx$$

(or) $$n = \frac{4\pi}{h^3}(2m_e^*)^{\frac{3}{2}} e^{(E_F-E_c)/kT} \int_{0}^{\infty} x^{\frac{1}{2}} e^{-x/kT}\, dx \qquad (8)$$

Using gamma function, $\int_{0}^{\infty} x^{\frac{1}{2}} e^{-x/kT}\, dx = \frac{(kT)^{\frac{3}{2}} \pi^{\frac{1}{2}}}{2}$ (9)

Substituting eqn (9) in (8), we get, $n = \frac{4\pi}{h^3}(2m_e^*)^{\frac{3}{2}} e^{(E_F-E_c)/kT} \times \frac{(kT)^{\frac{3}{2}} \pi^{\frac{1}{2}}}{2}$

(or) $$\boxed{n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{\frac{3}{2}} e^{(E_F-E_C)/kT}} \qquad (10)$$

This is the expression of electrons in the conduction band of an intrinsic semiconductor.

Concentration of holes in the Valence band

We know that if an electron is transferred from valence band to conduction band, a hole is created in valence band. Let *dp* be the number of holes in the valence band for the energy range *E* & *E*+*dE*. $\therefore dp = Z(E)\,(1 - F(E))\,dE$ (1)

Where Z (E) – Density of states in the energy range E & E+dE

(1 – F(E)) – Probability of unoccupied (vacant) electron state (presence of hole) in valence band

$$\therefore (1 - F(E))\,dE = 1 - \frac{1}{1+e^{(E-E_F)/kT}} = \frac{1+e^{(E-E_F)/kT}-1}{1+e^{(E-E_F)/kT}} = \frac{e^{(E-E_F)/kT}}{1+e^{(E-E_F)/kT}} \qquad (2)$$

$\because$ E < E_F in valence band, (E –E_F) has negative quantity & hence in denominator $e^{(E-E_F)/kT}$ is very small when compared with 1, hence $1+e^{(E-E_F)/kT} \cong 1$

$$\therefore 1 - F(E) = e^{(E-E_F)/kT} \qquad (3)$$

E_v is the top level in the valence band and having potential energy of a hole at rest. Hence

(E_v – *E*) is the kinetic energy of the hole at level below E_V. Hence,

Density of states in the valence band is $Z(E)dE = \frac{4\pi}{h^3}(2m_h^*)^{\frac{3}{2}}(E_V - E)^{\frac{1}{2}}\,dE$ (4)

Where, m_h^* - effective mass of the hole in the valence band

Substituting equation (4), (3) in (1), we get, $dp = \frac{4\pi}{h^3}(2m_h^*)^{\frac{3}{2}}(E_V - E)^{\frac{1}{2}}e^{(E-E_F)/kT}\,dE$ (5)

The number of holes in the valence band for the entire energy range is obtained by integrating

Above equation between the limits –α to E_V

i.e., $$\int dp = \int_{-\infty}^{E_V}\frac{4\pi}{h^3}(2m_h^*)^{\frac{3}{2}}(E_V - E)^{\frac{1}{2}}e^{(E-E_F)/kT}\,dE$$

(or) $$dp = \frac{4\pi}{h^3}(2m_h^*)^{\frac{3}{2}}e^{-E_F/kT}\int_{-\infty}^{E_V}(E_V - E)^{\frac{1}{2}}e^{E/kT}\,dE \qquad (6)$$

To solve this, assume the following:

$E_v - E = x$	$E = -\alpha$	$E = E_v$
$E = E_v - x$	$E_v - (-\alpha) = x$	$E_v - E_v = x$
$dE = -dx$	$x = \alpha$	$x = 0$

$$p = \frac{4\pi}{h^3}(2m_h^*)^{\frac{3}{2}} e^{-E_F/kT} \int_{\infty}^{0} x^{\frac{1}{2}} e^{-(x+E_v)/kT}(-dx)$$

(or) $$p = \frac{4\pi}{h^3}(2m_h^*)^{\frac{3}{2}} e^{(E_v-E_F)/kT} \int_{0}^{\infty} x^{\frac{1}{2}} e^{-x/kT}\,dx \qquad (7)$$

Using gamma function, $$\int_{0}^{\infty} x^{\frac{1}{2}} e^{-x/kT}\,dx = \frac{(kT)^{\frac{3}{2}}\pi^{\frac{1}{2}}}{2} \qquad (8)$$

Substituting eqn (8) in (7), we get, $$p = \frac{4\pi}{h^3}(2m_h^*)^{\frac{3}{2}} e^{(E_v-E_f)/kT} \times \frac{(kT)^{\frac{3}{2}}\pi^{\frac{1}{2}}}{2}$$

(or) $$\boxed{p = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{(E_v-E_F)/kT}} \qquad (9)$$

This is the expression of holes in the valence band of an intrinsic semiconductor

Intrinsic Carrier Concentration:

In intrinsic semiconductor, carrier concentration of electrons in conduction band (n_i) = carrier concentration of holes in valence band n_p & hence the intrinsic carrier concentration is $n_i^2 = n_i$ x n_p

$$n_i^2 = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{\frac{3}{2}} e^{(E_F-E_C)/kT} \times 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{(E_v-E_F)/kT} \qquad (1)$$

(or) $$n_i^2 = 4\left(\frac{2\pi kT}{h^2}\right)^{3} (m_e^* \times m_h^*)^{\frac{3}{2}} e^{(E_v-E_c)/kT}$$

(or) $$n_i^2 = 4\left(\frac{2\pi kT}{h^2}\right)^{3} (m_e^* \times m_h^*)^{\frac{3}{2}} e^{-E_g/kT} \qquad (2)$$

(or) $$n_i = 2\left(\frac{2\pi kT}{h^2}\right)^{\frac{3}{2}} (m_e^* \times m_h^*)^{\frac{3}{4}} e^{-\frac{E_g}{2kT}}$$

Where $E_c - E_v = E_g$ is the forbidden energy gap.

We know that, for an intrinsic semiconductor, the density of electron in the conduction band is equal to the density of holes in the valence band.

i.e., $n_i = n_p$ (3)

(or) $$2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{\frac{3}{2}} e^{(E_F-E_C)/kT} = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{(E_v-E_F)/kT}$$

Rearranging, we get, $$e^{(E_F-E_C)/kT} = \left(\frac{m_h^*}{m_e^*}\right)^{\frac{3}{2}} e^{(E_v-E_F)/kT}$$

(or) $$e^{(2E_F)/kT} = \left(\frac{m_h^*}{m_e^*}\right)^{\frac{3}{2}} e^{(E_v+E_c)/kT}$$

Taking log on both sides, we get

$$\log_e e^{(2E_F)/kT} = \log_e\left(\frac{m_h^*}{m_e^*}\right)^{\frac{3}{2}} + \log_e e^{(E_v+E_c)/kT}$$

(or) $$\frac{2E_F}{kT} = \frac{3}{2}\log_e\left(\frac{m_h^*}{m_e^*}\right)^{\frac{3}{2}} + \frac{E_v+E_c}{kT}$$

(or) $$E_F = \frac{kT}{2}\left[\frac{3}{2}\log_e\left(\frac{m_h^*}{m_e^*}\right) + \frac{E_v+E_c}{kT}\right] = \frac{3kT}{4}\log_e\left(\frac{m_h^*}{m_e^*}\right) + \frac{kT}{2}\left[\frac{E_v+E_c}{kT}\right]$$

(or) $$\boxed{E_F = \frac{3kT}{4}\log_e\left(\frac{m_h^*}{m_e^*}\right) + \left[\frac{E_v+E_c}{2}\right]}$$

(2)

If $m_e^* = m_h^*$, then $\log_e\left(\frac{m_h^*}{m_e^*}\right) = \log_e 1 = 0$,

hence the above equation becomes, $$E_F = \left[\frac{E_v+E_c}{2}\right]$$ (4)

Thus, the Fermi level is located half way between the top of the valence band and bottom of the conduction band. Its position is independent of temperature. If $m_e^* < m_h^*$,thus, the Fermi level is just above the middle of energy gap and its rises slightly with increasing temperature.

2.6 Extrinsic Semiconductors

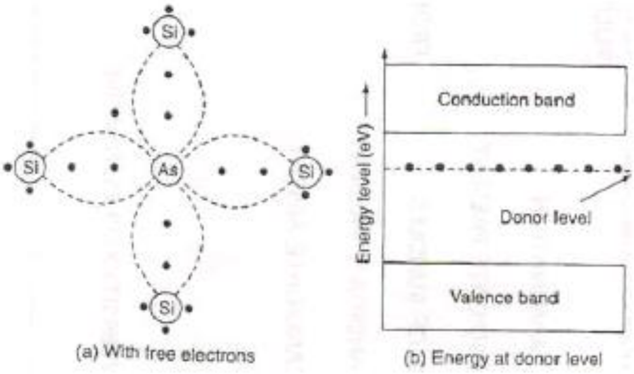
The application of intrinsic semiconductors is restricted due to its low conductivity. In electronic devices, high conducting semiconductors are more essential. The concentration of either electrons or holes in a semiconductor is increased depending upon the requirements in the electronic devices.

Extrinsic Semiconductors are classified into two categories based on the concentration of the charge carriers namely:

1. *n* - type semiconductors (electrons), and
2. *p* - type semiconductors (holes).

1. *n* - type Semiconductors

When a pentavalent atom such as arsenic (antimony, bismuth, phosphorus) is added as a dopant to the tetravalent silicon atom, the arsenic atom will occupy one site of the silicon atom. Thus, out of five free electrons in arsenic, four electrons make covalent bonds with the four neighbouring silicon atoms and the fifth one is loosely bound to the silicon atom, as shown in fig. 2



(a) With free electrons

(b) Energy at donor level

Fig. 2 Doping in *n*-type semiconductors.

The energy required to ionize the fifth electron is very less and hence, the thermal energy of the material shifts the free electron to the conduction band. Each arsenic atom contributes one free electron to the crystal and hence, it is called a donor impurity. In this type of semiconductor, the concentration of charge carriers (i.e., electrons) is more than that of holes. Therefore, these semiconductors are called n-type semiconductors. In an n-type semiconductor, electrons are the majority carriers while holes are the minority carriers.

Derivation

In *n* –type semiconductor, the donor level is just below the conduction band. N_d denotes the donor concentration & E_d represents the energy of the donor level.

Density of electrons per unit volume in the conduction band is given by

$$n = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{\frac{3}{2}} e^{(E_F - E_c)/kT} \qquad (1)$$

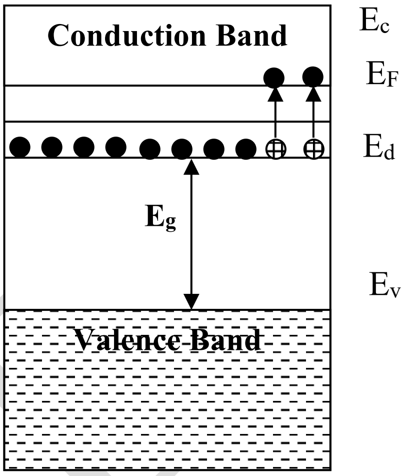
here E_f – Fermi energy; E_c – Energy corresponding to the bottom of the conduction band

Density of the ionized donors = $N_d\,[1 - F(E)]$

$$= N_d\left[1 - \frac{1}{1 + e^{(E_d - E_F)/kT}}\right]$$

$$= N_d\left[\frac{1 + e^{(E_d - E_F)/kT} - 1}{1 + e^{(E_d - E_F)/kT}}\right]$$

$$= N_d\left[\frac{e^{(E_d - E_F)/kT}}{1 + e^{(E_d - E_F)/kT}}\right]$$



Density of ionized donors $= \left[\frac{\dfrac{N_d}{1 + e^{(E_d - E_F)/kT}}}{e^{(E_d - E_F)/kT}}\right] = \left[\frac{N_d}{1 + e^{(E_F - E_d)/kT}}\right] \qquad (2)$

At equilibrium, the density of electrons in conduction band = Density of ionized donors.

Equating (1) & (2), $2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{\frac{3}{2}} e^{(E_F - E_c)/kT} = \frac{N_d}{1 + e^{(E_F - E_d)/kT}} \qquad (3)$

E_F lies more than few kT above donor levels, hence $e^{(E_d - E_F)/kT}$ is large compared to '1'. Hence the '1'' in denominator of R.H.S of equation (3) is neglected.

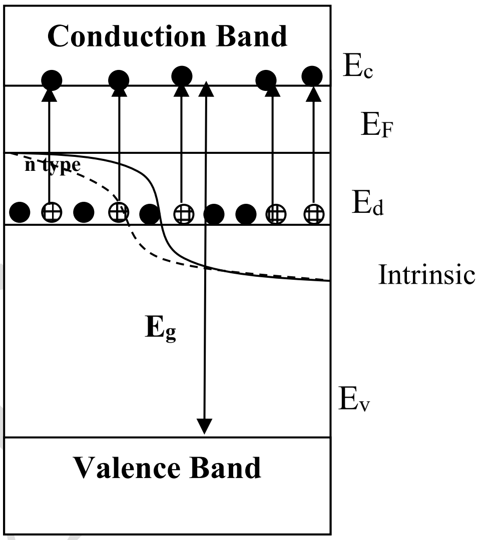
$$2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{\frac{3}{2}} e^{(E_F - E_c)/kT} = \frac{N_d}{e^{(E_F - E_d)/kT}}$$

(or) $2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{\frac{3}{2}} e^{(E_F - E_c)/kT} = N_d e^{(E_d - E_F)/kT} \qquad (4)$

Taking log on both sides,

$$\log\left[2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{\frac{3}{2}}\right] + \log_e e^{(E_F - E_c)/kT} = \log_e N_d e^{(E_d - E_F)/kT}$$

$$\text{(or)}\ \log\left[2\left[\frac{2\pi m_e^* kT}{h^2}\right]\right]^{\frac{3}{2}} + \frac{E_F - E_c}{kT} = \log N_d + \frac{E_d - E_F}{kT} \qquad (5)$$



$$\text{(or)}\ \frac{E_F - E_c - E_d + E_F}{kT} = \log N_d - \log\left[2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{\frac{3}{2}}\right]$$

$$\text{(or)}\ \frac{2E_F - E_c - E_d}{kT} = \log\left[\frac{N_d}{2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}}\right]$$

$$\text{(or)}\ 2E_F = E_d + E_c + kT\log\left[\frac{N_d}{2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}}\right]$$

$$\text{(or)}\ E_F = \frac{E_d + E_c}{2} + \frac{kT}{2}\log\left[\frac{N_d}{2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}}\right] \qquad (6)$$

Substituting the expression of E_F from (6) in (1), we get

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{\frac{3}{2}} \exp\left[\frac{\left(\frac{E_d + E_c}{2} + \frac{kT}{2}\log\left[\frac{N_d}{2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}}\right]\right) - E_c}{kT}\right] \qquad (7)$$

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{\frac{3}{2}} \exp\left[\left(\frac{E_d + E_c - 2E_c}{2kT} + \frac{1}{2}\log\left[\frac{N_d}{\left[2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}\right]}\right]\right)\right]$$

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{\frac{3}{2}} \exp\left[\left(\frac{E_d - E_c}{2kT} + \log\left[\frac{(N_d)^{1/2}}{\left[2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}\right]^{1/2}}\right]\right)\right]$$

$$n = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{\frac{3}{2}} \frac{\left[\frac{N_d}{2}\right]^{1/2}}{\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/4}} e^{(E_d - E_c)/2kT} \qquad (8)$$

Rearranging the expression (8), we have

(or) $n = (2N_d)^{1/2}\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/4} e^{(E_d - E_c)/2kT}$

(or) $n = (2N_d)^{1/2}\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/4} e^{(-\Delta E)/2kT}$

Where ▲E= E_c - E_d is the ionization energy to transfer Energy from donor energy level to the conduction band.

Results:

- The density of electrons is proportional to square root of donor concentration and valid at low temperature alone
- At high temperature, intrinsic carrier concentration must take along with this concentration

Variation of Fermi level with temperature and impurity concentration

Fermi level of n – type semiconductor is $E_F = \frac{E_d + E_c}{2} + \frac{kT}{2}\log\left[\frac{N_d}{2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}}\right]$ (1)

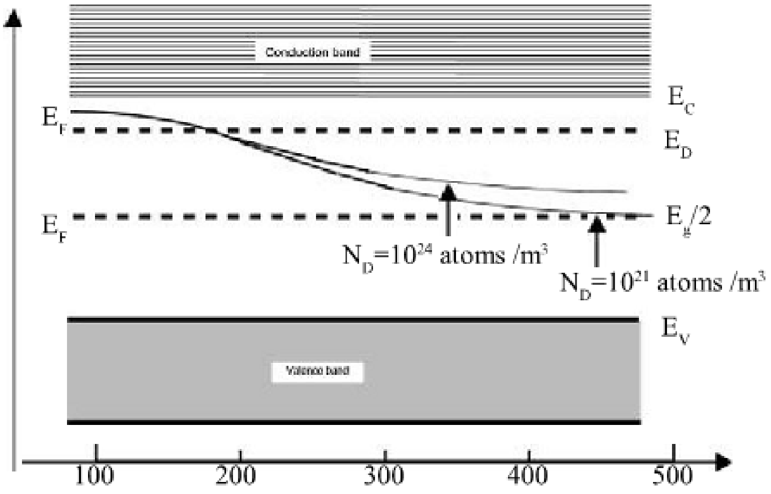
At T = 0K, the above equation reduces to $E_F = \frac{E_d + E_c}{2}$ (2)

(i) At 0K, Fermi level lies exactly at the centre of the donor level and bottom of the conduction band.

(ii) As the temperature is gradually increased from a low temperature, the contribution of electron increases and at very high temperature, it far exceeds the donor concentration and the intrinsic behaviour predominates at higher temperature.

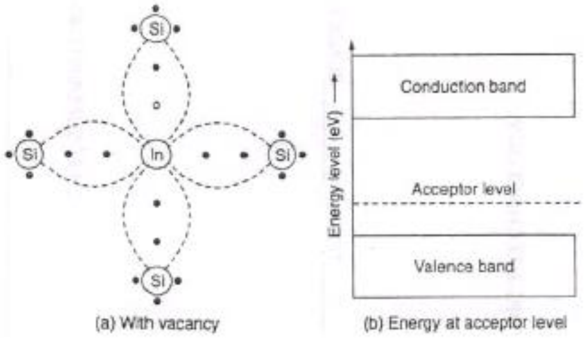
(iii) Fermi level shifts downwards when the temperature is increased and finally reaches the middle of the band gap (or) intrinsic Fermi level.

(iv) Further when the concentration of donors increases, the extrinsic behaviour also extends up to very high temperature & Fermi level reaches the middle of the band gap only at high temperature.



2. *p* -type Semiconductors

Instead of a pentavalent atom, the addition of a trivalent atom indium (In) to the tetravalent silicon atom, occupies the crystal site of the silicon atom as shown in Fig.3



(a) With vacancy (b) Energy at acceptor level

Fig.3

The three valence electrons in indium make covalent bands with the three neighbouring silicon atoms, whereas the fourth bond has an empty space known as hole due to the deficiency of one electron. Therefore, when a trivalent atom is added to silicon, it creates a hole in the valence band. The dopant (indium) accepts an electron from the neighbouring silicon atom to form a covalent bond and hence, it is called an acceptor. The hole in the valence band moves freely and hence, the current flows through the material.

This type of electrical conduction will take place only when the dopant valency is less than that of the parent atom. Such semiconductors are called p – type semiconductors. In a p-type semiconductor, holes are the majority current carriers and electrons are the minority current carriers.

In p - type semiconductor, the acceptor energy level is just above the valence band. Let E_a represents the energy of the acceptor level and N_a represents the number of acceptor atoms per unit volume.

Density of holes per unit volume in valence band is given by $p = 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{\frac{3}{2}} e^{(E_v - E_F)/kT}$ (1)

Where E_v is the energy corresponding to the top of the conduction band

Density of the ionized acceptors = $N_a F(E_a) = \frac{N_a}{1 + e^{(E_a - E_F)/kT}}$ (2)

Since $E_a - E_F$ is very large when compared to kT. $e^{(E_a - E_F)/kT}$ is a large quantity and thus '1' in denominator of R.H.S of equation(2) is neglected. hence, equation(2), reduces to

$$N_a F(E_a) = \frac{N_a}{e^{(E_a - E_F)/kT}} = N_a e^{(E_F - E_a)/kT} \quad (3)$$

At equilibrium, the density of holes in Valence band = Density of ionized acceptors

$$2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{\frac{3}{2}} e^{(E_v - E_F)/kT} = N_a e^{(E_F - E_a)/kT} \quad (4)$$

Taking log on both sides,

$$\log\left[2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{\frac{3}{2}} e^{(E_v - E_F)/kT}\right] = \log\left[N_a e^{(E_F - E_a)/kT}\right]$$

$$\text{(or)}\ \log\left[2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{\frac{3}{2}}\right] + \frac{E_v - E_F}{kT} = \log N_a + \frac{E_F - E_a}{kT}$$

(5)

Rearranging the expression (5), we have

$$\frac{E_F - E_a - E_v + E_F}{kT} = -\log N_a + \log\left[2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{\frac{3}{2}}\right]$$

(or) $$\frac{2E_F-(E_a+E_v)}{kT}=-\log_e\left\{\frac{N_a}{\left[2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{\frac{3}{2}}\right]}\right\}$$

(or)

$$2E_F=(E_a+E_v)-kT\log_e\left\{\frac{N_a}{\left[2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{\frac{3}{2}}\right]}\right\}$$

(or)

$$E_F=\frac{(E_a+E_v)}{2}-\frac{kT}{2}\log_e\left\{\frac{N_a}{\left[2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{\frac{3}{2}}\right]}\right\} \qquad (6)$$

Sub. The expression of E_F in equation (6), we get

$$p=2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}}\exp\left[\frac{\left(E_v-\frac{E_v-E_a}{2}+\frac{kT}{2}\log\left[\frac{N_a}{2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2}}\right]\right)_c}{kT}\right] \qquad (7)$$

(or) $$p=2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}}\exp\left[\left(\frac{2E_v-E_v-E_a}{2kT}+\frac{1}{2}\log\left[\frac{N_a}{2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2}}\right]\right)\right] \qquad (8)$$

(or) $$p=2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}}\frac{\left[\frac{N_a}{2}\right]^{\frac{1}{2}}}{\left[\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2}\right]^{\frac{1}{2}}}e^{\frac{E_v-E_a}{2kT}}$$

(or) $$p=(2N_a)^{1/2}\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4}e^{(E_v-E_a)/2kT} \qquad (9)$$

If $E_a - E_v = \blacktriangle E$ is the acceptor ionization energy required to move the electron from valence band to acceptor energy level, then equation (9) becomes,

$$p = (2N_a)^{1/2}\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4} e^{-\Delta E/2kT} \qquad (10)$$

Results:

- Density of holes in valence band is proportional to square root of acceptor concentration
- At very high temperature p – type semiconductor behaves like an intrinsic semiconductor

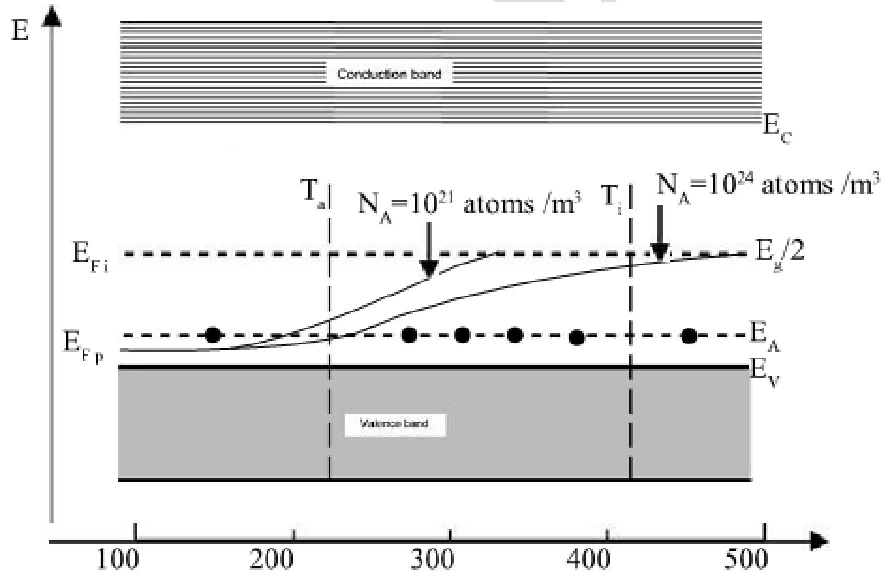
Variation of Fermi level with temperature

We know that

$$E_F = \frac{(E_a + E_v)}{2} - \frac{kT}{2}\log_e\left\{\frac{N_a}{\left[2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{\frac{3}{2}}\right]}\right\} \qquad (1)$$

At T = 0K , the above expression becomes, $E_F = \dfrac{E_a + E_v}{2}$ (2)

At 0K the Fermi level lies exactly halfway between acceptor level E_a and top of the valence band E_v



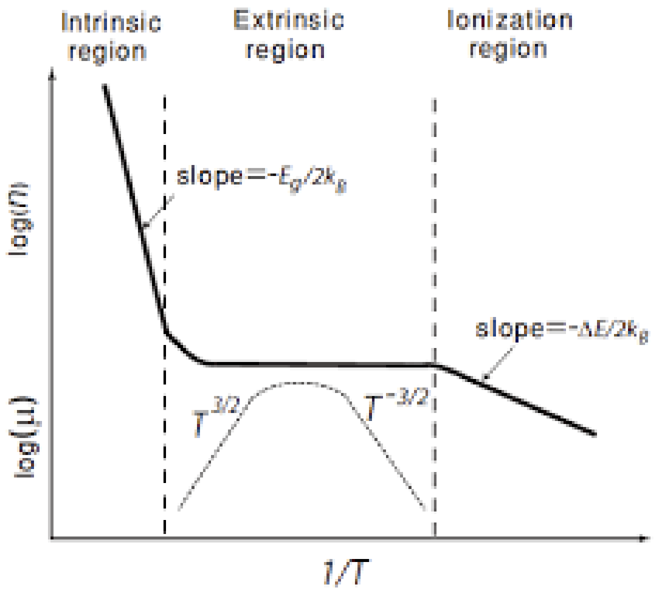
As the temperature increases, the Fermi level shifts upwards, at a particular temperature, when all the acceptor atoms are ionized and Fermi level crosses the acceptor level

At very high temperature, the Fermi level is shifted to intrinsic Fermi level and behave as intrinsic semiconductor.

2.7. Variation of carrier concentration with temperature and impurity.

In extrinsic semiconductor, the resistivity decreases linearly with increase in temperature. This variation is considered under three different regions.

- Ionization or impurity range
- Exhaust or extrinsic range
- Intrinsic range



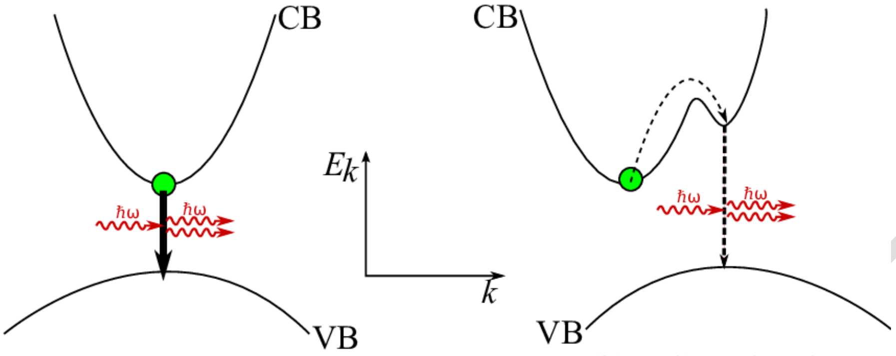
For a *n*-type semiconductor, the variation of carrier concentration *n* and *p* with temperature is shown in figure. At 0 K, both conduction and valence bands are free from any charge carriers and hence, the electrical conductivity is zero. With increase in temperature, the donor atoms gets ionised and hence electron concentration in conduction band increases with temperature until all the donor atoms are ionised. This range is known as **impurity / ionization** region. When the temperature is further increased to room temperature, there are no more donor atoms to be ionised and hence the concentration of electrons in conduction band remains constant over a certain temperature range. This region is known as **exhaust or extrinsic region.** As the temperature is increased further, the electrons in valence band are lifted across the forbidden gap to conduction ban. Thus, electron concentration increases in conduction band considerably. With further increase in temperature, more and more electrons from valence band reach conduction band and completely outnumber the donor electrons. The material practically becomes intrinsic and so this range is called **intrinsic** range.

2.8. Direct and Indirect bandgap semiconductors

Semiconductors also classified as (i) direct bandgap semiconductor and (ii) indirect bandgap semiconductor. The electrons and holes in the semiconductor have energy and momentum. The momentum (k) depends on the energy (E_k). A plot of E_k versus k is shown in figure. The lower curve represent energy and momentum values of holes in valence band of semiconductor. Similarly upper curves denote corresponding values for electrons in conduction band.

In direct bandgap semiconductor, the energy maximum of valence band and energy minimum of conduction band are having same momentum value.

During the recombination of electron from CB with hole in VB, the momentum of the electron remains virtually constant. The energy equal to bandgap energy is released as **light photon.**



(a) Direct bandgap (b) Indirect bandgap

But, in case of indirect bandgap semiconductor, the maximum energy of valence band and minimum energy of conduction band are having different values of momentum. During recombination, electron first loses momentum such that it has momentum equal to the momentum corresponding to energy maximum of valence band. To converse the momentum, emission of third particle known as a phonon is generated. Thus, in this type of recombination phonon is generated.

2.9. Degenerate and non-degenerate Semiconductors

In a direct band gap semiconductor, the top of the valence band and the bottom of the conduction band occur at the same value of momentum. In an indirect band gap semiconductor, the maximum energy of the valence band occurs at a different value of momentum to the minimum in the conduction band energy. Typical dopant concentrations are in range of ppm (parts of gas per million) or ppb and form individual energy levels in the band gap. This is because N_D and N_A are much smaller than the effective density of states at the band edges (Nc and Nv). These are called non-degenerate semiconductors. Under such conditions, it is possible to consider the dopants as individual atoms in the Si lattice and ignore interactions between the dopant levels. This is why dopants are shown as individual energy levels, close to the valence or conduction band, and not as an energy band. As the dopant concentration increases, the individual energy levels start to overlap so that there are no longer energy levels but energy bands. This happens at typical dopant concentrations of 10^{19} and 10^{20} cm $^{-3}$, comparable to N_c and N_v (10^{20} cm $^{-3}$). These are called degenerate semiconductors, and their energy band diagram is shown schematically in figure 1. The dopant energy levels can merge with the conduction or valence band, so that the Fermi energy lies within the band. Thus, degenerate semiconductors behave more like metals than semiconductors. Degenerate doped semiconductors are used for some opto-electronic devices like lasers due to the large carrier concentration.

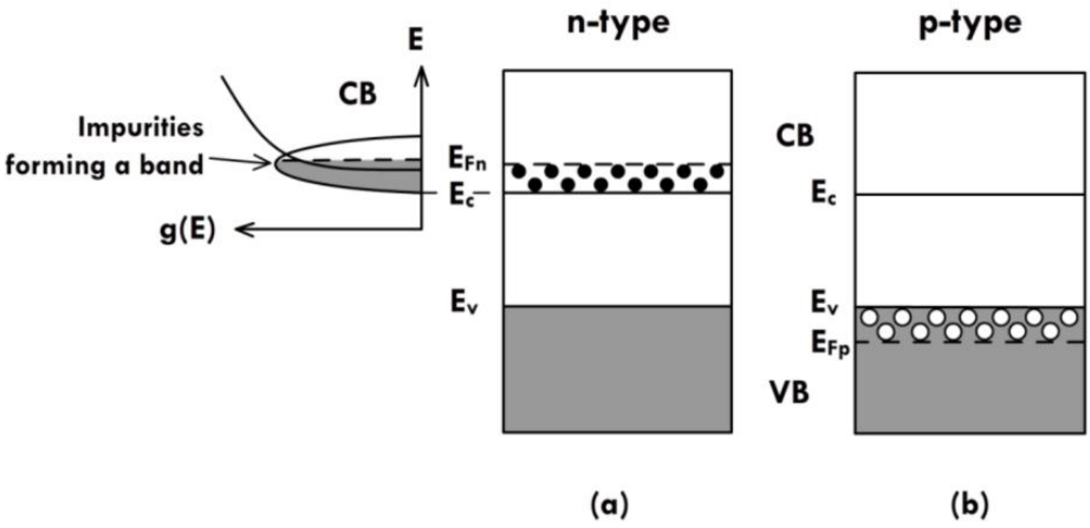
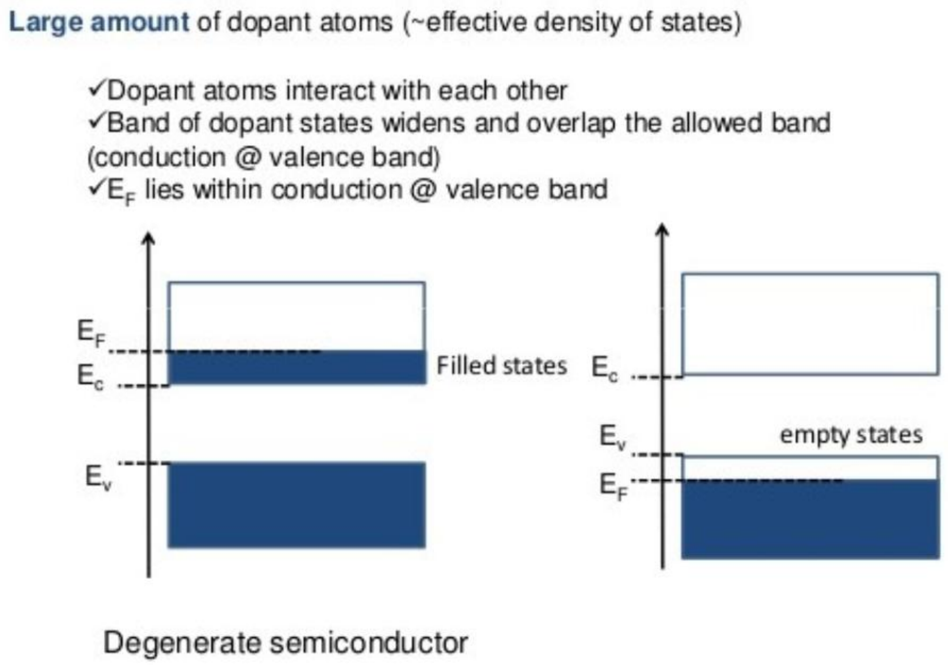


Figure 1 Degenerate (a) n and (b) p type semiconductors. The impurities form an energy band that can merge with either the valence or conduction band. The Fermi level lies within this band rather that in the band gap. Degenerate semiconductors hence have properties similar to metals.

A **degenerate semiconductor** is a semiconductor with such a high level of doping that the material starts to act more like a metal than as a semiconductor.

What happens if we add dopants at much higher concentrations? Dopant atoms come much closer to each other and it is no longer valid to assume the donor levels as atom like. If the inter-atomic distance is closer (typically < 10nm) then the atomic levels turn into bands. This leads to significant changes in the crystal structure as well as the physical properties. Another very important effect is, highly doped semiconductors come to freeze-out at much lower temperatures, meaning the freeze-out region is almost eliminated. Such highly doped semiconductors are called Degenerate semiconductors.


Large amount of dopant atoms (~effective density of states)

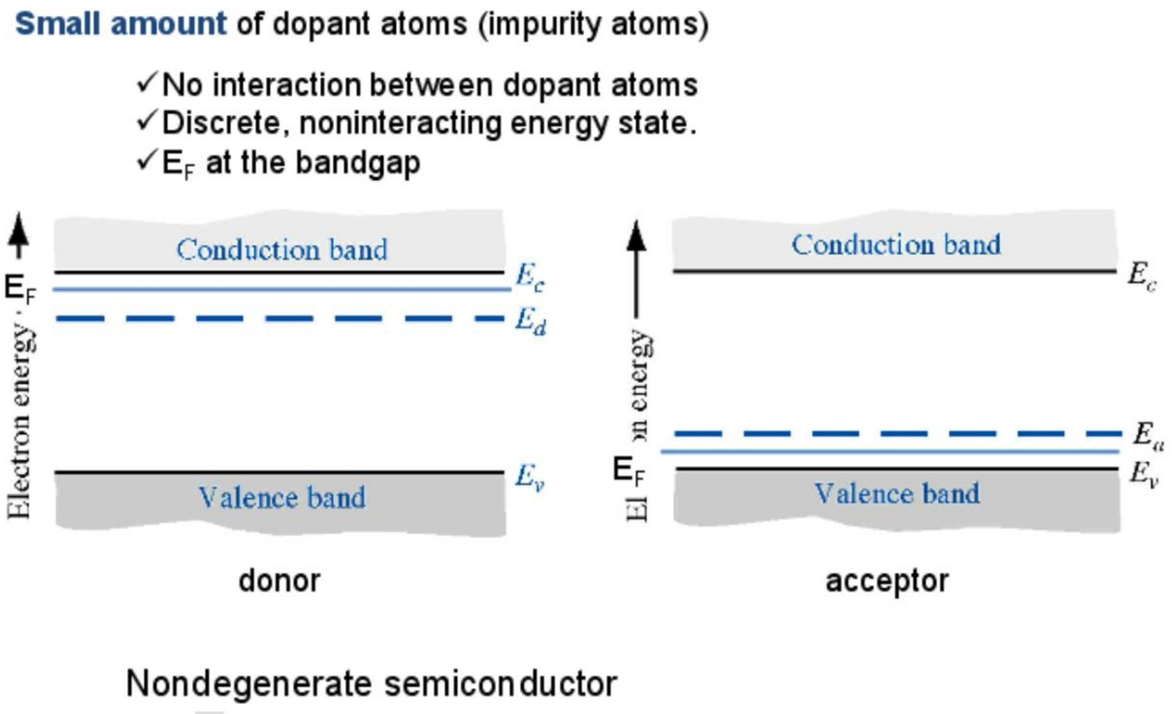
✓Dopant atoms interact with each other
✓Band of dopant states widens and overlap the allowed band (conduction @ valence band)
✓E_F lies within conduction @ valence band

Degenerate semiconductor

At moderate doping levels the dopant atoms create individual doping levels that can often be considered as localized states that can donate electrons or holes by thermal promotion (or an optical transition) to the conduction or valence bands respectively. At high enough impurity concentrations the individual impurity atoms may become close enough neighbors that their doping levels merge into an impurity band and the behavior of such a system ceases to show the typical traits of a semiconductor, e.g. its increase in conductivity with temperature. On the other hand a degenerate semiconductor still has far fewer charge carriers than a true metal so that its behavior is in many ways intermediary between semiconductor and metal.

Many copper chalcogenides are degenerate p-type semiconductors with relatively large numbers of holes in their valence band. An example is the system $LaCuOS_{1-x}Se_x$ with Mg doping. It is a wide gap p-type degenerate semiconductor. The hole concentration does not change with temperature, a typical trait of degenerate semiconductors.

It is also to be noted that the donors (or acceptors) energy levels are assumed as atom like. Such assumptions are limited up to a certain level of dopant concentration, and such extrinsic semiconductors are called **non-degenerate** semiconductors.



Small amount of dopant atoms (impurity atoms)

✓No interaction between dopant atoms
✓Discrete, noninteracting energy state.
✓E_F at the bandgap

Nondegenerate semiconductor

Non-degenerate semiconductors are defined as semiconductors for which the Fermi energy is at least $3kT$ away from either band edge. The key to a semiconductor is that the Fermi level is somewhere between the conduction and valence bands, in the forbidden gap. There are no available states within a few kT of the Fermi level. This is a non-degenerate semiconductor

2.10 Hall effect

"*When the conductor carrying a current (I) is placed in a perpendicular magnetic field (B), a potential difference is developed inside the conductor in a direction normal to the directions of both the current and magnetic field*"

This phenomenon is known as Hall Effect and the corresponding voltage thus generated is called Hall voltage

Explanation

Consider an external field applied along the X-axis of the specimen. Assuming that the material is *n*-type semiconductor, the current flow consists mainly of electrons moving from right to left, corresponding to the conventional current direction.

When this specimen is placed in a magnetic field 'B' and if '*v*' is the velocity of the electrons perpendicular to the magnetic field then each one of them will a experience a downward force of magnitude Be*v*

This downward force (Lorentz Force F_L) due to magnetic field causes the electrons to be deflected in the downward direction and hence there is an accumulation of negative charges on the bottom face of the slab. This causes the bottom face of the slab to be more negative with respect to the top face and a potential difference is established from top to bottom of the specimen. This potential difference causes a field E_H called Hall field in negative y direction. There is a force eE_H acting on the electron in the upward direction due to this field.

Theory of Hall Effect

At equilibrium, the downward force Bev will balance the upward force eE_H

$$Bev = eE_H \qquad (1)$$

In a uniform sample, the electric current density (J) is related to the drift velocity as

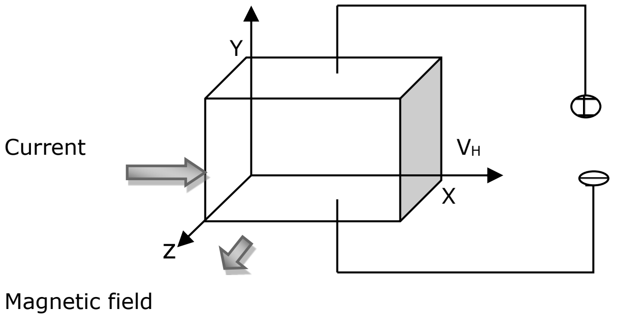
$$J = -neV$$

Where n is the concentration of electrons.

$$V = \frac{-J}{ne} \qquad (2)$$

Substituting equation (2) in (1),

$$E_H = \frac{-BJ}{ne}$$



This can be written as $E_H = BJR_H$ (4)

Where R_H = - 1 / ne is called Hall coefficient

The negative sign indicates that the developed field is in the negative y direction.

III^{rly}, the Hall coefficient for p – type semiconductor is R_H = 1 / p e.

(top) +++++
(n-Type)
--------------(bottom)

LIED

(top) ---------
(p-Type)
++++++++(bottom)

Where p is the concentration of holes

Determination of Hall coefficient

The hall field per unit current density per unit magnetic induction is defined as hall coefficient.

If t is the thickness of the sample and V_H is the hall voltage, then $V_H = E_H\, t$ (5)

Where E_H is hall field.

From equation (4), we get $E_H = R_H J_x B$

Substituting the value of (5) in above equation, we get $V_H = R_H\, J_x\, B\, t$ (6)

Now the current density J_x can be written as $J_x = \frac{I_x}{bt}$ (7)

Where 'b ' is the width and bt is the area of cross section of the sample

Substituting equation (7) in equation (6), we get $V_H = \frac{R_H I_X B t}{bt}$

$$V_H = \frac{R_H I_X B}{b} \qquad (8)$$

$$\text{(or)} \quad \boxed{R_H = \frac{V_H b}{I_X B}} \qquad (9)$$

For an n – type semiconductor $\boxed{R_H = \frac{-V_H b}{I_X B}}$ (10)

Mobility of charge carriers

We know that hall coefficient $R_H = \frac{-1}{ne}$

This expression is correct only when the charge carriers is free from any attractive force in energy band and moves with constant drift velocity. But this is not true in the case of semiconductors.

Considering the average speed, it is shown that $R_H = \frac{-1.18}{ne}$ for electrons and $R_H = \frac{1.18}{pe}$ for holes.

We know that the electrical conductivity and mobility is related by $\sigma = n\, e\, \mu_e$

(or) $\mu_e = \frac{\sigma}{ne}$ & hence $\mu_e = \frac{\sigma_e}{ne}$ and hence $\mu_e = \frac{-R_H \sigma_e}{1.18}$ (11)

Similarly $\mu_h = \frac{-R_H \sigma_h}{1.18}$ (12)

Experimental Determination of Hall Coefficient:

The experimental setup for the measurement of Hall voltage is shown in figure.

A semiconducting material is taken in the form of a rectangular slab of thickness *'t'* and breadth *'b'*. A suitable current I_x ampere is allowed to pass through this sample along the X axis by connecting it to battery

The sample is placed between the poles pieces of an electromagnet such that the applied magnetic field coincides with the z – axis.

Hall voltage (V_H) which is developed in the sample is measured by fixing two probes at the centers of the bottom and top faces of the sample.

By measuring Hall voltage, Hall coefficient is calculated from the formula $R_H = \frac{-V_H b}{I_x B}$

Applications

(i) The sign of the hall coefficient is used to determine whether a give semiconductor is *n* – type or *p* – type

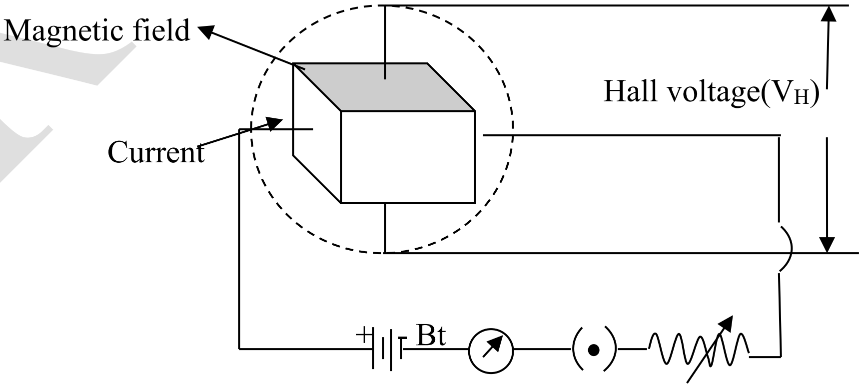
(ii) Once Hall coefficient R_H is measured, the carrier concentration can be determined from

$n = 1 / e R_H$

(iii) The mobility of charge carriers can be obtained if conductivity is known. $\mu_e = \sigma_e R_H$

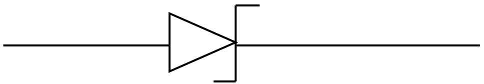
(iv) Hall voltage V_H for a given current is proportional to *B*. Hence measurement of V_H measures the magnetic field *B*.

(v) This instrument gives an output proportional to the product of two signals. Thus if current *I* is made proportional to one input and if *B* is made proportional to the other input, then the Hall voltage V_H is proportional to the product of the two inputs.



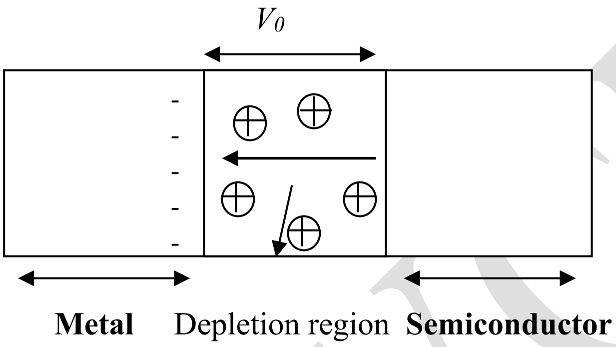
2.10 Schottky diode

It is the junction formed between a metal and n – type semiconductor. When the metal has a higher work function than that of n – type semiconductor then the junction formed is called Schottky diode. The Fermi level of the semiconductor is higher (since its work function is lower) than the metal. Figure shows Schottky diode and its circuit symbol.

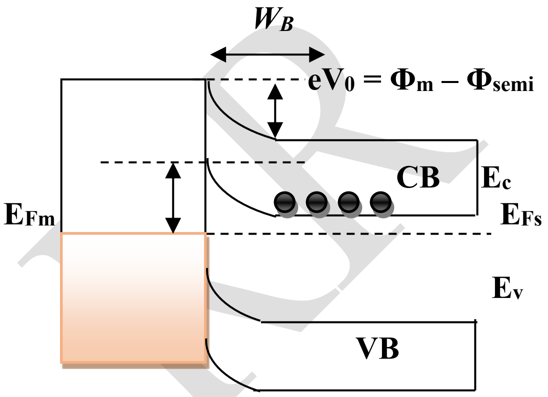


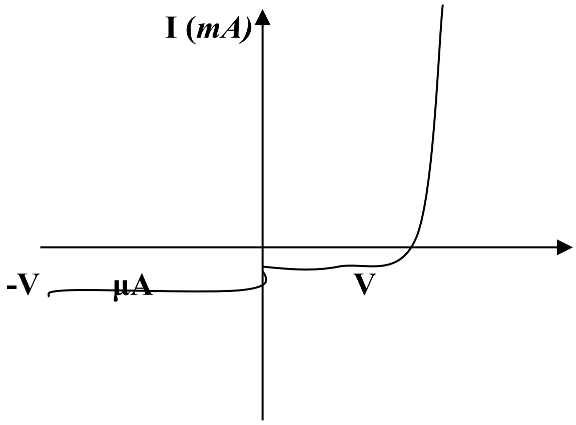
The electrons in the conduction level of the semiconductor move to the empty energy states above the fermi level of the metal. This leaves a positive charge on the semiconductor side and a negative charge (due to the excess electrons) on the metal sie as shown in figure. This leads to a contact potential.

When a Schottky junction is formed between metal and semiconductor, fermi level lines up. Also a positive potential is formed on the semiconductor side. The formation of a depletion region of width W_D within the semiconductor is shown in figure. Because the depletion region extends within a certain depth in the semiconductor, there is bending of the energy bands on the semiconductor side. *Band* bend up in the direction of the electric field produced in depletion region. There is a built in potential V_o in the Schottky junction. From the figure this is given by the difference in work functions $eV_0 = \varphi_m - \varphi_{semi}$



Energy band diagram





Working

The behaviour of Schottky diode is further studied by forward and reverse bias.

(a)Forward Bias

In this bias, metal is connected to positive terminal and n – type semiconductor is connected to negative terminal of the battery. In the forward biased Schottky junction, the external potential opposes the in- built potential. The electrons injected from the external circuit into the *n – type* semiconductor have a lower barrier to overcome before reaching the metal. This leads to a current in the circuit which increases with increasing external potential.

(b) Reverse Bias

In reverse bias, metal is connected to negative terminal and $n-type$ semiconductor to positive terminal of the battery. In the case of reverse bias, the external potential is applied in the same direction as the junction potential. This increases the width of depletion region further and hence there is no flow of electron from semiconductor to metal. So Schottky junction acts as rectifier. i.e., it conducts in forward bias but not in reverse bias.

$V-I$ Characteristics

The $V-I$ characteristics of the junction is shown in figure. There is an exponential increase in current in the forward bias while there is a very small current in reverse bias.

Advantages

It has very low capacitance
It will immediately switch from ON to OFF state (fast recovery time)
Applying a small voltage is enough to produce large current
It has high efficiency
It operates at high frequencies
It produces less noise.

2.11 Ohmic contact

An ohmic contact is a type of metal semiconductor junction. It is formed by a contact of a metal with a heavily doped semiconductor. When the semiconductor has a higher work function than that of metal, then the junction formed is called the ohmic junction.

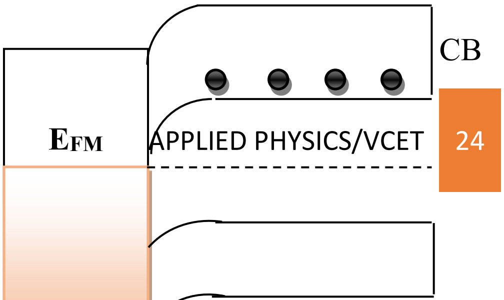
Here, the current is conducted equally in both directions and there is a very little voltage drop across the junction. Before contact, fermi levels of the metal and semiconductor are at different positions as shown in figure.

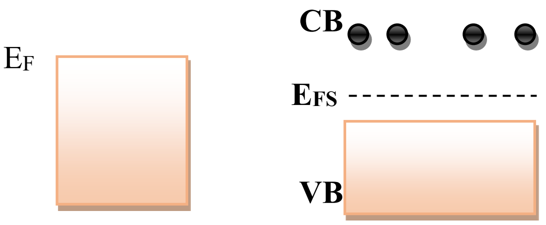
Working

After contact, the ohmic junction is shown in figure. At equilibrium, the electrons move from the metal to the empty states in the conduction band of semiconductor. Thus, there is an accumulation region near the interface (on the semiconductor side). The accumulation region has higher conductivity than the bulk semiconductor due to this higher concentration of electrons. Thus, a ohmic contact behaves as a resistor conducting in both forward and reverse bias. The resistivity is determined by the bulk resistivity of the semiconductor.

$V-I$ Characteristics

The *V-I* characteristics of the ohmic contact is shown in figure. The current is directly proportional to the potential across the junction and it is symmetric about the origin, as shown in figure. Thus, ohmic contacts are non-rectifying and show negligible voltage drop and resistance irrespective of the direction and magnitude of current.

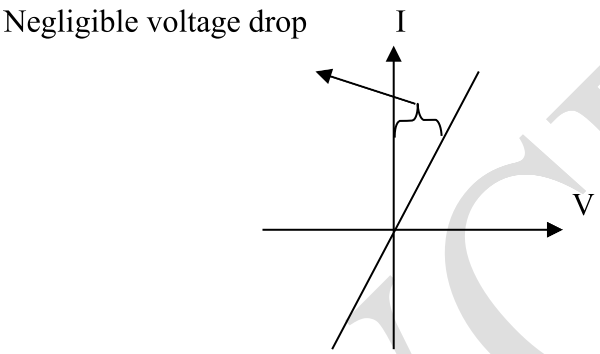




E_{FS}

VB

Metal Semiconductor Metal Semiconductor



Applications

The use of ohmic contacts is to connect one semiconductor device to another, an IC, or to connect an IC to its external terminals.